Application Report
SNOA373C–October 1996–Revised April 2013

OA-28 Low-Sensitivity, Bandpass Filter Design With Tuning Method

ABSTRACT

This application report covers the design of a Sallen-Key bandpass biquad. It gives a design with low component and op amp sensitivities. Then it gives a filter tuning method to compensate for parasitics. A design example illustrates these methods. These biquads are also called KRC or VCVS [voltage-controlled, voltage-source].

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1 Introduction

Changes in component values over process, environment and time affect the performance of a filter. To achieve a greater production yield, the filter needs to be insensitive to these changes. This application report presents a design algorithm that results in low sensitivity to component variation. For information on evaluating the sensitivity performance of your filter, see [6].

To achieve the best production yield, the nominal filter design must also compensate for component and board parasitics. This document gives a method to empirically tune your filter. For the background theory, see [5], [7].

2 Filter Tuning Overview

This section shows a simple tuning method that compensates for the parasitic elements in your filter.

To minimize the impact of parasitics:

- Keep signal paths as short as possible
- Minimize the length of all feedback loops
- Use components with small parasitics
- Use good PCB layout techniques
- Use an op amp with adequate bandwidth ($f_{3\ dB}$) and slew rate (SR):
  - $f_{3\ dB} \geq 10\ f_H$
  - $SR > 5\ f_H\ V_{PEAK}$
  - where f_H is the highest frequency in the passband of the filter, and V_{PEAK} is the largest peak voltage. Make sure the op amp is stable at the chosen gain.

To compensate for the parasitic elements:

1. Start with a low sensitivity, low parasitic design
2. Calculate the sensitivities of the filter response parameters to the resistors and capacitors [6]
3. Measure the filter's response. The important parameters to extract are:
   - Maximum passband gain (H_p)
   - Pole frequency (ω_p)
   - Pole quality (Q_p)
     - The *Design Example* section gives a simple extraction method. Use accurate component values for the prototype filter so that the nominal design point will be near the center of the possible component values
4. Use the information in steps 2 and 3 to adjust the resistor and capacitor values:
   - Set up the linear equations relating the relative change in filter response parameters ($\Delta H_p/H_p$, $\Delta\omega_p/\omega_p$ and $\Delta Q_p/Q_p$) to the relative change in the components to be adjusted
   - The number of components to change is the same as the number of filter response parameters
   - The coefficients of these linear equations are the component sensitivities [6]
   - Solve for the relative change in component values
   - Calculate the new component values
5. Repeat steps 3 and 4 until the nominal response is close enough to the desired response.

3 KRC Bandpass Biquad Design

The biquad shown in Figure 1 is a Sallen-Key bandpass biquad. V_{IN} needs to be a voltage source with low output impedance.

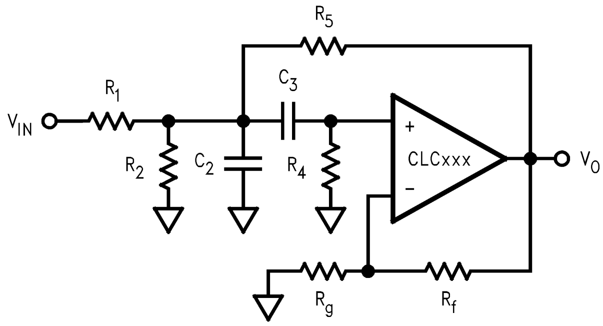

Figure 1. Bandpass Biquad

R_2 attenuates the input signal for low gains. V_{IN}, R_1 and R_2 can be replaced with their Thévenin equivalent voltage (αV_{IN}) and impedance (R_{12}):

$\alpha = R_2/(R_1 + R_2)$

$R_{12} = R_1 \parallel R_2$

The transfer function is:

$$\frac{V_O}{V_{IN}} \approx \frac{H_p(\omega_p/Q_p)s}{s^2 + (\omega_p/Q_p)s + (\omega_p^2)}, \; s = j\omega \tag{1}$$

where:

$$\omega_p/Q_p = \left(\frac{1}{R_{12}} + \frac{1}{R_4} - \frac{K-1}{R_5}\right) \cdot \frac{1}{C_2} + \frac{1}{R_4 C_3}$$

$$\omega_p^2 = \frac{1}{R_4 C_2 C_3}\left(\frac{1}{R_{12}} + \frac{1}{R_5}\right)$$

$$K = 1 + R_f/R_g$$

$$H_p = \frac{\alpha K}{R_{12} C_2} \cdot \frac{Q_p}{\omega_p} \tag{2}$$

To achieve low sensitivities, use this design algorithm:

1. Use this biquad when: $0.5 \le Q_p < 5.0$Steps 2 and 3 assume this condition to be true.
2. Partition the gain:
   - Use a low noise amplifier before this biquad if you need a large gain
   - Initialize the peak passband gain in 1 of 3 ways:
   - For the best sensitivity performance, use:$H_p \approx 1.0$
   - For reasonable sensitivity performance and reduced component spreads, use:$H_p \approx \max\{1.0, Q_p\}$
   - For dynamic range performance, scale H_p as needed. Limit the peak gain to:$H_p < 10.0$
3. Set the input attenuation:$\alpha = \min\{1.0, H_p\}$
4. Initialize one of the resistor spreads ($r^2 = R_{12}/R_4$) and the op amp gain (K):
   - $A_1 = 0.0381Q_p^{1.51}(H_p/\alpha)^{-127}$
   - $A_2 = 0.00206Q_p^{-1.92}(H_p/\alpha)^{1.39}$
   - $r^2 \approx \max\{0.1, A_1 + A_2\}$
   - $B_1 = 0.456(\max\{1, Q_p\})^{-1.22}(H_p/\alpha)^{1.22}$
   - $B_2 = 0.0260(\max\{1, Q_p\})^{1.76}(H_p/\alpha)^{-1.51}$
   - $K \approx 1.0 + \max\{0.1, B_1 + B_2\}$
5. Select an op amp with adequate bandwidth ($f_{3\ dB}$) and slew rate (SR):
   - $f_{3\ dB} \ge 10\ f_H$
   - $SR > 5\ f_H\ V_{PEAK}$

- where f_H is the highest frequency in the passband, and V_{PEAK} is the largest peak voltage. Make sure the op amp is stable at a gain of $A_v = K$.

6. For current-feedback op amps, use the recommended value of R_f for a gain of $A_v = K$. For voltage-feedback op amps, select R_f for noise and distortion performance. Then set R_g for the correct gain:$R_g = R_f/K - 1)$
7. Calculate the capacitor spread ($c^2 = C_2/C_3$), and the other resistor spread ($\beta^2 = R_{12}/R_5$):

$$A_0 = (K - 1)(\alpha K Q_p / H_p)^2$$

$$A_1 = r^2 + K(1 - \alpha/H_p)$$

$$c^2 = \frac{1}{r^2} \cdot \frac{2A_0}{A_1 + \sqrt{A_1^2 + 4A_0}}$$

$$\beta^2 = \left(\frac{\alpha K Q_p}{H_p}\right)^2 \left(\frac{1}{c^2 r^2}\right) - 1$$

8. Initialize the resistance level $(R = \sqrt{R_{12} R_4})$.Increasing R will:
   - Increase the output noise
   - Improve the distortion performance
   - Improve the isolation between the op amp outputs and C_2 and C_3
   - Make the parasitic capacitances a larger fraction of C_2 and C_3
9. Calculate the capacitance level $(C = \sqrt{C_2 C_3})$:$C = \sqrt{1 + \beta^2}/(\omega_p R)$
10. Calculate the resistors and capacitors:
    - $R_{12} = rR$
    - $R_1 = R_{12}/\alpha$
    - $R_2 = R_{12}/(1 - \alpha)$
    - $R_4 = R/r$
    - $R_5 = R_{12}/\beta^2$
    - $C_2 = cC$
    - $C_3 = C/c$
11. Set the resistors and capacitors to the nearest standard values.

The component sensitivity formulas are in the table below. The sensitivities to $\alpha_i = K$ are a measure of this biquad's sensitivity to the op amp group delay [5]. To evaluate this biquad's sensitivity performance, see [6]. To manually pre-distort this filter, and compensate for parasitic capacitances, see [5], [7].

α_i	$S^{H_p}_{\alpha_i}$	$S^{\omega_p}_{\alpha_i}$	$S^{Q_p}_{\alpha_i}$
R_1	$\frac{H_p}{K} - 1$	$\frac{-\alpha}{2(1 + \beta^2)}$	$S^{H_p}_{R_1} + S^{\omega_p}_{R_1} + 1$
R_2	$\frac{(1 - \alpha)H_p}{\alpha K}$	$\frac{-(1 - \alpha)}{2(1 + \beta^2)}$	$S^{H_p}_{R_2} + S^{\omega_p}_{R_2}$
R_4	$\frac{H_p(1 + c^2)r^2}{\alpha K}$	$-\frac{1}{2}$	$S^{H_p}_{R_4} + S^{\omega_p}_{R_4}$
R_5	$\frac{-H_p(K - 1)\beta^2}{\alpha K}$	$\frac{-\beta^2}{2(1 + \beta^2)}$	$S^{H_p}_{R_5} + S^{\omega_p}_{R_5}$
C_2	$\frac{-H_p c^2 r^2}{\alpha K}$	$-\frac{1}{2}$	$S^{H_p}_{C_2} + S^{\omega_p}_{C_2} + 1$
C_3	$-S^{H_p}_{C_2}$	$-\frac{1}{2}$	$-S^{Q_p}_{C_2}$
R_f	$\left(\frac{H_p\beta^2}{\alpha} + 1\right) \cdot \frac{K - 1}{K}$	0	$S^{H_p}_{R_f} + S^{\omega_p}_{R_f} - \frac{K - 1}{K}$
R_g	$-S^{H_p}_{R_f}$	0	$-S^{Q_p}_{R_f}$
K	1	0	$\frac{H_p\beta^2}{\alpha}$

(3)

4 KRC Bandpass Biquad Tuning Method

To tune this filter, use this algorithm:

1. Start with a low-sensitivity design.
2. Calculate the sensitivities of H_p, ω_p and Q_p to the components.
3. Set up the linear equations:
   - Choose the 3 components α_i that will be changed to adjust H_P, ω_p and Q_p
   - Create this sensitivity matrix using the formulas (see *Appendix A* for a simple method that uses measurement or simulation results): $$M_3 = \begin{bmatrix} S_{\alpha_1}^{H_p} & S_{\alpha_2}^{H_p} & S_{\alpha_3}^{H_p} \\ S_{\alpha_1}^{\omega_p} & S_{\alpha_2}^{\omega_p} & S_{\alpha_3}^{\omega_p} \\ S_{\alpha_1}^{Q_p} & S_{\alpha_2}^{Q_p} & S_{\alpha_3}^{Q_p} \end{bmatrix}$$
   - Invert the sensitivity matrix (M_3^{-1})
4. Measure the filter response, and then extract H_p, ω_p and Q_p:
   - Find the maximum gain magnitude: $H_p = \max\{|H(j\omega)|\}$
   - Find the −3 dB corner frequencies f_1 and f_2, where $f_2 > f_1$ $$\omega_p = 2\pi\sqrt{f_1 f_2}$$
   - Calculate ω_p and Q_p: $Q_p = \sqrt{f_1 f_2}/(f_2 - f_1)$
5. Calculate the needed changes in H_p, ω_p and Q_p (X): $\Delta X/X = 1 - X_{meas}/X_{nom}$ where X_{nom} and X_{meas} are the nominal and measured values of X. Limit the relative changes in X:
   $$\Delta X/X \leftarrow \max\{-0.5, \min\{1.0, \Delta X/X\}\} \qquad (4)$$
6. Calculate the needed component values: $$\begin{bmatrix} \Delta\alpha_1/\alpha_1 \\ \Delta\alpha_2/\alpha_2 \\ \Delta\alpha_3/\alpha_3 \end{bmatrix} = M_3^{-1} \cdot \begin{bmatrix} \Delta H_p/H_p \\ \Delta\omega_p/\omega_p \\ \Delta Q_p/Q_p \end{bmatrix}$$
   - Estimate the relative changes in α_i, and then limit them: $\frac{\Delta\alpha_i}{\alpha_i} \leftarrow \max\left\{-0.5, \min\left\{1.0, \frac{\Delta\alpha_i}{\alpha_i}\right\}\right\}$
   - Calculate the new α_i: $\alpha_i \leftarrow \alpha_i(1 + \Delta\alpha_i/\alpha_i)$
   - Change the filter components to these new values; *use accurate component values when building the prototype filter* so that the nominal design point will be near the center of the possible component values
7. Repeat steps 4–6 until the nominal response is close enough to the desired response.

5 Design Example

The circuit shown in Figure 2 is a 4th-order bandpass filter. This filter cascades two bandpass biquads: sections A and B. Use a voltage source with low output impedance, such as the CLC111 buffer, for V_{IN}.

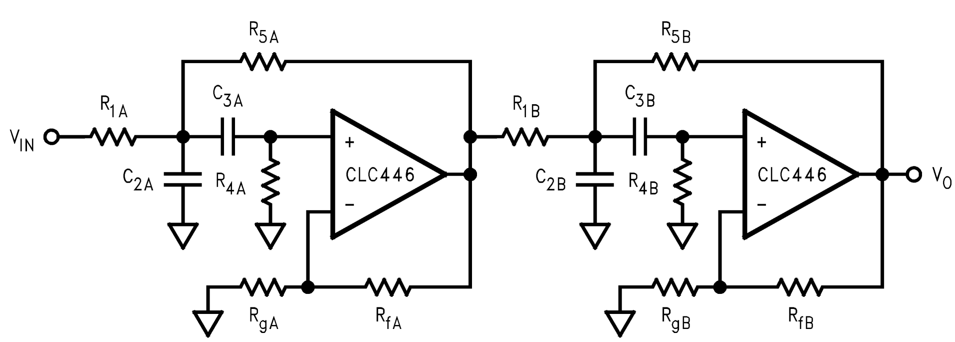

Figure 2. Bandpass Filter

The nominal filter specifications are:

f_{sl} = 15 MHz—(lower stopband edge frequency)

f_{cl} = 40 MHz—(lower passband edge frequency)

f_{cu} = 60 MHz—(upper passband edge frequency)

f_{su} = 135 MHz—(upper stopband edge frequency)

A_p = 3.0 dB—(maximum passband ripple)

A_s = 30 dB—(minimum stopband attenuation)

H_p = 0 dB—(passband voltage gain)

The second-order Butterworth lowpass prototype filter meets these specifications [1] through [4]. The H_p values shown below give a maximum gain of 1.00 from V_{IN} to each biquad output. The transformed filter is:

Section		A	B
$\omega_p/2\pi$	[MHz]	42.36	56.65
Q_p	[ ]	3.501	3.501
H_p	[V/V]	1.000	2.043

Overall Design:

1. Restrict the resistor and capacitor ratios to:$0.1 \le c^2, r^2, \beta^2 \le 10$
2. Use 1% resistors (chip metal film, 1206 SMD)
3. Use 5% capacitors (ceramic chip, 1206 SMD)
4. Use standard resistor and capacitor values
5. Use the same H_p in both sections to simplify the design. Also set the overall gain to 1.00:$H_p = \sqrt{(1.000)(2.043)} = 1.429$

Section A Design:

1. Q_p (3.501) meets the required limits
2. H_p (1.429) is between the first two criteria in step 2 of the design algorithm; the sensitivity performance and component spreads should be reasonable
3. Initialize α to 1.00; R_{2A} is an open circuit
4. Initialize r^2 & K:
   - $A_1 = 0.1606$ $A_2 = 0.0003$
   - $r^2 = 0.1609$
   - $B_1 = 0.1528$ $B_2 = 0.1376$
   - $K = 1.290$
5. The CLC446 is a current-feedback op amp:
   - $f_{3\ dB} = 400$ MHz < 10 f_H = 600 MHz ($f_H = f_{cu}$); the op amp strongly affects the filter
   - SR = 2000 V/µs, while a 60 MHz, 2 V_{pp} sinusoid requires more than 300 V/µs
6. Set R_{fA} to the CLC446's recommended R_f at A_V = +1.290, then calculate R_{gA}:
   - $R_{fA} = 392\Omega$
   - $R_{gA} = R_{fA}/(K - 1) = 1352\Omega$
7. Calculate c^2 and β^2:
   - $A_0 = 2.897$ $A_1 = 0.5482$
   - $c^2 = 9.011$
   - $\beta^2 = 5.889$

8. Initialize R = 300Ω
9. Calculate C: $C = \dfrac{\sqrt{1 + 5.889}}{2\pi(42.36\,\text{MHz}) \cdot (300\,\Omega)} = 32.87\,\text{pF}$
10. The initial values are in the table below

Section B Design:

H_p and Q_p are the same as in section A, but ω_p is different. To change the pole frequency, scale the resistors R_{1B}, R_{4B} and R_{5B}:

$$R_{xB} \leftarrow R_{xA} \bullet (\omega_{pA}/\omega_{pB}) = R_{xA} \bullet 0.7477$$

The initial component values are:

Component		Initial Value	
		Section A	Section B
R_1	[Ω]	120	90.0
R_4	[Ω]	748	559
R_5	[Ω]	20.4	15.3
C_2	[pF]	98.7	98.7
C_3	[pF]	11.0	11.0
R_f	[Ω]	392	392
R_g	[Ω]	1352	1352

Filter Tuning:

This section uses simulated results; different layout and component parasitics will change the tuning results. Simulations used the following parasitics:

- 0.2 pF across all resistors
- 1.0 pF to ground at CLC446 non-inverting inputs
- A group delay of 0.56 ns for the CLC446 at 50 MHz, using a good simulation model

1. The sensitivities for sections A and B are equal since they are not functions of ω_p. They are:

α_i	$S^{H_p}_{\alpha i}$	$S^{\omega_p}_{\alpha i}$	$S^{Q_p}_{\alpha i}$
R_1	0.11	−0.07	1.04
R_4	1.78	−0.50	1.28
R_5	−1.89	−0.43	−2.32
C_2	−1.61	−0.50	−1.11
C_3	1.61	−0.50	1.11
R_f	2.12	0.00	1.89
R_g	−2.12	0.00	−1.89
K	1.00	0.00	8.42

2. To tune the filter, change R_1, R_4 and R_g:

$$\begin{bmatrix} \Delta R_1/R_1 \\ \Delta R_4/R_4 \\ \Delta R_g/R_g \end{bmatrix} = M_3^{-1} \bullet \begin{bmatrix} \Delta H_p/H_p \\ \Delta\omega_p/\omega_p \\ \Delta Q_p/Q_p \end{bmatrix}$$

where:

$$M_3 = \begin{bmatrix} 0.11 & 1.78 & -2.12 \\ -0.07 & -0.50 & 0.00 \\ 1.04 & 1.28 & -1.89 \end{bmatrix}$$

$$M_3^{-1} = \begin{bmatrix} -0.91 & -0.62 & 1.02 \\ 0.13 & -1.91 & -0.14 \\ -0.41 & -1.64 & -0.07 \end{bmatrix}$$

3. The results of tuning section A are:

Iteration No		1	2	3	4
R_1	[Ω]	120	98.7	89.3	90.8
R_4	[Ω]	748	496	488	492
R_g	[Ω]	1352	676	708	700
H_p	[V/V]	0.736	1.625	1.373	1.433
$\omega_p/2\pi$	[MHz]	34.62	41.76	42.57	42.32
Q_p	[ ]	2.212	4.226	3.335	3.504
$\Delta H_p/H_p$	[%]	48.5	−13.7	3.92	−0.28
$\Delta\omega_p/\omega_p$	[%]	18.3	1.42	−0.50	0.09
$\Delta Q_p/Q_p$	[%]	36.8	−20.7	4.74	−0.09
$\Delta R_1/R_1$	[%]	−17.9	−9.52	1.58	—
$\Delta R_4/R_4$	[%]	−33.8	−1.59	0.79	—
$\Delta R_g/R_g$	[%]	−50.0	4.75	−1.13	—

4. The results of tuning section B are:

Iteration No		1	2	3	4
R_1	[Ω]	90.0	66.5	62.9	63.3
R_4	[Ω]	559	363	357	360
R_g	[Ω]	1352	740	784	779
H_p	[V/V]	0.993	1.663	1.384	1.428
$\omega_p/2\pi$	[MHz]	45.66	55.94	56.95	56.62
Q_p	[ ]	3.029	4.174	3.391	3.496
$\Delta H_p/H_p$	[%]	30.5	−16.4	3.15	0.07
$\Delta\omega_p/\omega_p$	[%]	19.4	1.25	−0.53	0.05
$\Delta Q_p/Q_p$	[%]	13.5	−19.2	3.14	0.14
$\Delta R_1/R_1$	[%]	−26.1	−5.48	0.67	—
$\Delta R_4/R_4$	[%]	−35.0	−1.83	0.98	—
$\Delta R_g/R_g$	[%]	−45.3	6.00	−0.64	—

Figures 3 and 4 show the simulated filter gain. The curve numbers are:

1. The ideal gain
2. The gain for the initial design (Iteration 1)
3. The gain for the tuned filter (Iteration 4)

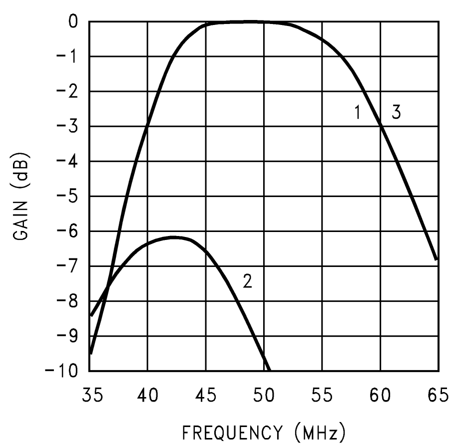

Figure 3. Simulated Filter Magnitude Response

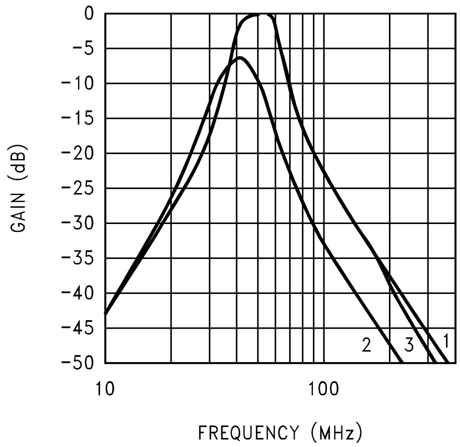

Figure 4. Simulated Filter Magnitude Response

The final standard component values are:

Component		Standard Tuned Value	
		Section A	Section B
R_1	[Ω]	90.9	63.4
R_4	[Ω]	487	357
R_5	[Ω]	20.5	15.4
C_2	[pF]	100	100
C_3	[pF]	11	11
R_f	[Ω]	392	392
R_g	[Ω]	698	787

6 SPICE Models

SPICE models are available for most of Comlinear's amplifiers. These models support nominal DC, AC, AC noise and transient simulations at room temperature.

We recommend simulating with Comlinear's SPICE models to:

- Predict the op amp's influence on filter response
- Support quicker design cycles

Include board and component parasitic models to obtain a more accurate prediction of the filter's response.

To verify your simulations, we recommend bread-boarding your circuit.

7 Summary

This application report contains an easy to use design algorithm for a low sensitivity, Sallen-Key bandpass biquad. Designing for low H_p, ω_p and Q_p sensitivities gives:

- Reduced filter variation over process, temperature and time
- Higher manufacturing yield
- Lower component cost

A low sensitivity design is not enough to produce high manufacturing yields. This document shows how to tune the filter to compensate for parasitics; no assumptions about the parasitics are necessary. The components must also have low tolerance, small parasitics and low temperature coefficients.

8 References

1. R. Schaumann, M. Ghausi and K. Laker, *Design of Analog Filters: Passive, Active RC, and Switched Capacitor.* New Jersey: Prentice Hall, 1990.
2. A. Zverev, *Handbook of FILTER SYNTHESIS*. John Wiley & Sons, 1967.
3. A. Williams and F. Taylor, *Electronic Filter Design Handbook*, McGraw Hill, 1995.
4. S. Natarajan, *Theory and Design of Linear Active Networks*, Macmillan, 1987.
5. *OA-21 Component Pre-Distortion for Sallen Key Filters* (SNOA369)
6. *OA-27 Low-Sensitivity, Lowpass Filter Design* (SNOA372)
7. *OA-29 Low-Sensitivity, Highpass Filter Design With Parasitic Compensation* (SNOA374)

NOTE: The circuits included in this application report have been tested with National Semiconductor parts that may have been obsoleted and/or replaced with newer products. To find the appropriate replacement part for the obsolete device, see the *CLC to LMH Conversion Table* (SNOA428).

Appendix A Estimating the Sensitivity Matrix

For filters where the sensitivity formulas are not readily available, this appendix gives a simple method to estimate the entries in the sensitivity matrix.

To estimate the sensitivity matrix entries using measurement or simulation results, use this algorithm:

1. Choose the 3 components α_i that will be changed to adjust H_p, ω_p and Q_p (X)
2. Calculate the sensitivities of H_p, ω_p and Q_p to the chosen components:
   - Extract the parameters H_p, ω_p and Q_p at the nominal values of α_i
   - Extract the parameters H_p, ω_p and Q_p when only one α_i is different from its nominal value; this results in 3 sets of 3 modified performance parameters
   - Estimate the sensitivities (X is H_p, ω_p or Q_p):

$$S_{\alpha_1}^{X} \approx \frac{\Delta X}{X} \cdot \frac{\alpha_1}{\Delta\alpha_1}\bigg|_{\Delta\alpha_2 = \Delta\alpha_3 = 0}$$

$$S_{\alpha_2}^{X} \approx \frac{\Delta X}{X} \cdot \frac{\alpha_2}{\Delta\alpha_2}\bigg|_{\Delta\alpha_1 = \Delta\alpha_3 = 0}$$

$$S_{\alpha_3}^{X} \approx \frac{\Delta X}{X} \cdot \frac{\alpha_3}{\Delta\alpha_3}\bigg|_{\Delta\alpha_1 = \Delta\alpha_2 = 0}$$

3. From the sensitivity matrix:

$$M_3 = \begin{bmatrix} S_{\alpha_1}^{H_p} & S_{\alpha_2}^{H_p} & S_{\alpha_3}^{H_p} \\ S_{\alpha_1}^{\omega_p} & S_{\alpha_2}^{\omega_p} & S_{\alpha_3}^{\omega_p} \\ S_{\alpha_1}^{Q_p} & S_{\alpha_2}^{Q_p} & S_{\alpha_3}^{Q_p} \end{bmatrix}$$

4. Invert the sensitivity matrix (M_3^{-1})

Example:

As an example, suppose that the following measurements result from the 4 conditions in Step 2 (italicized numbers are changed from nominal):

Condition No		**1**	**2**	**3**	**4**
R_1	[Ω]	120	*115*	120	120
R_4	[Ω]	748	748	*715*	748
R_g	[Ω]	1352	1352	1352	*1300*
H_p	[V/V]	0.736	*0.729*	*0.681*	*0.792*
$\omega_p/2\pi$	[MHz]	34.62	*34.84*	*35.51*	*34.54*
Q_p	[ ]	2.212	*2.110*	*2.096*	*2.368*

where the Condition # means:

1. Nominal values
2. $\Delta R_1 \neq 0$, and $\Delta R_2 = \Delta R_3 = 0$
3. $\Delta R_2 \neq 0$, and $\Delta R_1 = \Delta R_3 = 0$
4. $\Delta R_3 \neq 0$, and $\Delta R_1 = \Delta R_2 = 0$

The sensitivity of H_p to R_1 is estimated as:

$$S_{R_1}^{H_p} = \frac{0.729 - 0.736}{0.736} \cdot \frac{120}{115 - 120} \approx 0.23 \tag{5}$$

Estimating the other sensitivities produces these sensitivity matrices:

$$M_3 \approx \begin{bmatrix} 0.23 & 1.69 & -1.98 \\ -0.15 & -0.58 & 0.06 \\ 1.11 & 1.19 & -1.83 \end{bmatrix}$$

$$M_3^{-1} \approx \begin{bmatrix} -0.95 & -0.70 & 1.00 \\ 0.20 & -1.70 & -0.27 \\ -0.45 & -1.53 & -0.11 \end{bmatrix} \quad (6)$$

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