Operational amplifier gain stability, Part 2: DC gain-error analysis

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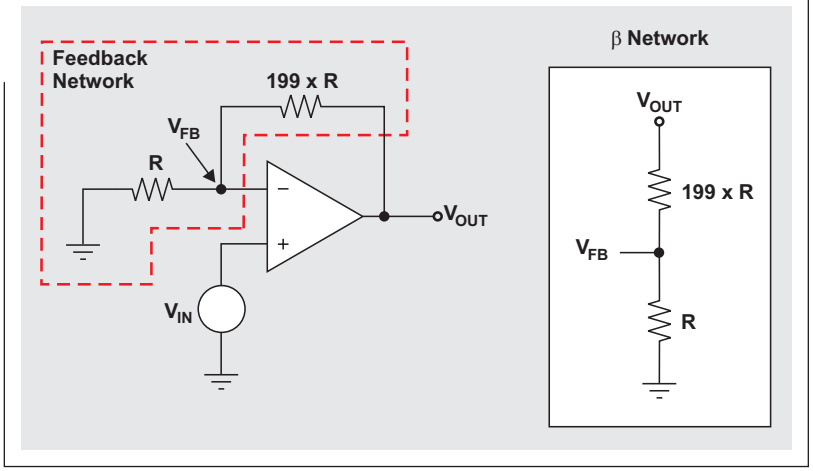
Introduction

The goal of this three-part series of articles is to provide readers with an in-depth understanding of gain accuracy in closed-loop circuits using two of the most common operational amplifier (op amp) configurations: non-inverting and inverting. Often, the effects of various op amp parameters on the accuracy of the circuit's closed-loop gain are overlooked and cause an unexpected gain error both in the DC and AC domains.

This article, Part 2, focuses on DC gain error, which is primarily caused by the finite DC open-loop gain of the op amp as well as its temperature dependency. This article builds upon the results obtained in Part 1 (see Reference 1), in which two separate equations were derived for calculating the transfer functions of non-inverting and inverting op amps. Part 2 presents a step-by-step example of how to calculate the worst-case gain error, starting with finding the pertinent data from the product data sheet. It then shows how to use the data in conjunction with the two aforementioned equations to perform the gain-error calculation.

In Part 3, the gain error for AC input signals will be calculated. In the AC domain, the closed-loop gain error is affected by the AC open-loop response of the op amp. Part 3 will discuss one of the most common mistakes that occur when the AC gain response is calculated.

Figure 1. Non-inverting op amp configuration with ideal closed-loop gain of +200



Transfer functions of non-inverting and inverting op amps

In Part 1 (Reference 1), the closed-loop transfer function of the non-inverting op amp configuration in the frequency domain was calculated. Specifically, the transfer function was derived with the assumption that the op amp had a first-order open-loop response. For calculating gain error, the magnitude response is of interest. For convenience, the result is repeated in Equation 1:

$$\left|A_{CL}(f)\right|_{dB} = 20\log\frac{\dfrac{A_{OL_DC}}{1+\beta\times A_{OL_DC}}}{\sqrt{1+\dfrac{f^2}{f_0^2}\times\dfrac{1}{(1+\beta\times A_{OL_DC})^2}}}, \quad (1)$$

where β is defined as

$$\beta = \frac{V_{FB}}{V_{OUT}} = \frac{R_I}{R_I + R_F}. \quad (2)$$

Also derived in the same article was the equation for calculating the magnitude of the inverting configuration's closed-loop gain. The result is repeated in Equation 3:

$$\left|A_{CL}(f)\right|_{dB} = 20\log\frac{\alpha\dfrac{A_{OL_DC}}{1+\beta\times A_{OL_DC}}}{\sqrt{1+\dfrac{f^2}{f_0^2}\times\dfrac{1}{(1+\beta\times A_{OL_DC})^2}}} \quad (3)$$

Equation 3 uses the same variable β defined by Equation 2. Additionally, the variable α is defined by Equation 4:

$$\alpha = \frac{V_{FB}}{V_{IN}} = \frac{R_F}{R_I + R_F} \quad (4)$$

At this point, the closed-loop gain for non-inverting and inverting amplifiers is represented by Equations 1 and 3, respectively. These equations will be used for subsequent analysis. The analysis of DC closed-loop circuits has been treated in slightly different ways in References 2 to 7; however, the results agree with this analysis.

DC gain error for non-inverting configuration

To illustrate the impact of an op amp's finite open-loop gain on the accuracy of DC closed-loop gain in a non-inverting configuration, a step-by-step example will be presented on how to calculate the gain error when the op amp is set in an ideal closed-loop gain. An ideal closed-loop gain of 200 (1/β = 200), as shown in Figure 1, will be used. This example focuses on using only the Texas Instruments (TI)

OPA211 op amp, but circuit designers can repeat the calculation with similar values from the data sheet of any other op amp they choose.

To calculate the DC closed-loop-gain error of a non-inverting op amp, Equation 1 is evaluated for zero frequency (f = 0 Hz):

$$A_{CL_DC} = A_{CL}(0\ Hz) = \frac{A_{OL_DC}}{1+\beta \times A_{OL_DC}} \quad (5)$$

In the case of an ideal op amp with infinite open-loop gain, the DC closed-loop gain of the non-inverting configuration is reduced to

$$A_{CL_DC(ideal)} = \lim_{A_{OL_DC}\to\infty} \frac{A_{OL_DC}}{1+\beta \times A_{OL_DC}} = \frac{1}{\beta}. \quad (6)$$

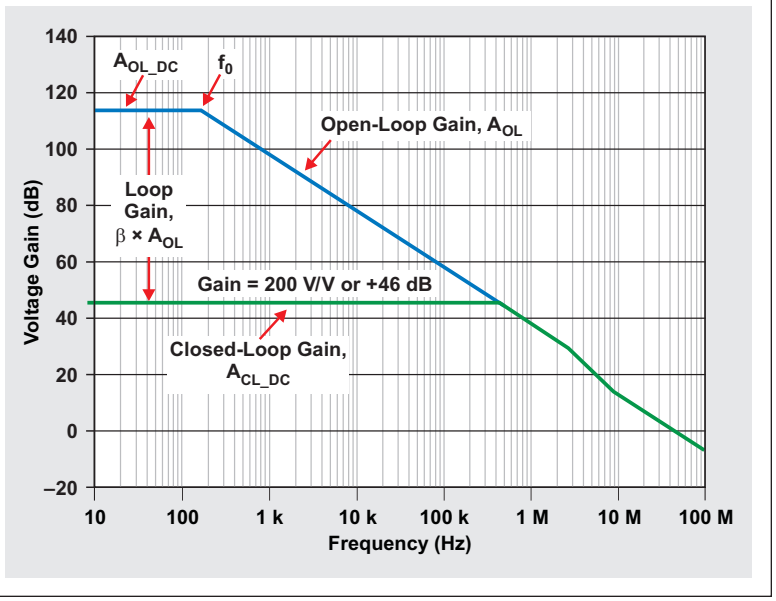
In other words, the DC closed-loop gain is entirely determined by the external feedback network.

From the closed-loop models of non-inverting and inverting amplifiers in Figures 3 and 6, respectively, in Part 1 (see Reference 1), it can be seen that the open-loop gain of the op amp is the ratio of V_{OUT} to the input-error voltage, V_{ERR}. V_{ERR} is the voltage difference between the inverting and non-inverting op amp inputs. It can also be seen as input offset voltage. In a product data sheet, the open-loop gain is typically expressed in decibels. In this case, the number represents the ratio of V_{OUT} to V_{ERR} in the logarithmic domain. For future calculation, A_{OL_DC} must always be converted from decibels to V/V. As an example, an op amp with an open-loop gain of 106 dB can be written in terms of V/V as

$$A_{OL_DC}\big|_{V/V} = 10^{\frac{A_{OL_DC}\big|_{dB}}{20}} = 10^{\frac{106\ dB}{20}} = \frac{V_{OUT}}{V_{ERR}} = 199{,}526\ \frac{V}{V}. \quad (7)$$

Figure 2 shows the simplified open-loop gain of the OPA211 along with the closed-loop gain in a non-inverting configuration. The difference between these two curves is the loop gain, $\beta \times A_{OL}$. Because the focus of this example is DC gain error, only the loop gain at low frequency ($\beta \times A_{OL_DC}$) is of interest.

Figure 2. OPA211's simplified open-loop and closed-loop gain curves



When using the data from the typical curves, designers should consider possible variations. To calculate worst-case values, the open-loop-gain data provided in the product data sheet should be used. Such data are shown in Table 1 for the TI OPA211/2211 op amps. As the table shows, when the output signal is more than 200 mV from the supply rails and has a 10-kΩ load, the typical value for the DC open-loop gain is 130 dB, while the minimum ensured gain is 114 dB. To calculate the typical and the worst-case DC gain

Table 1. Excerpt from TI OPA211/2211 data sheet

ELECTRICAL CHARACTERISTICS: V_S = ±2.25V to ±18V

BOLDFACE limits apply over the specified temperature range, T_A = –40°C to +125°C.

At T_A = +25°C, R_L = 10kΩ connected to midsupply, V_{CM} = V_{OUT} = midsupply, unless otherwise noted.

PARAMETER		CONDITIONS	Standard Grade OPA211AI, OPA2211AI			High Grade OPA211I			UNIT
			MIN	TYP	MAX	MIN	TYP	MAX	
OPEN-LOOP GAIN									
Open-Loop Voltage Gain	**A_{OL}**	**(V–) + 0.2V ≤ V_O ≤ (V+) – 0.2V, R_L = 10kΩ**	**114**	**130**		**114**	**130**		**dB**
	A_{OL}	(V–) + 0.6V ≤ V_O ≤ (V+) – 0.6V, R_L = 600Ω	110	114		110	114		dB
Over Temperature									
OPA211	**A_{OL}**	**(V–) + 0.6V ≤ V_O ≤ (V+) – 0.6V, I_O ≤ 15mA**	**110**			**110**			**dB**
OPA211	**A_{OL}**	**(V–) + 0.6V ≤ V_O ≤ (V+) – 0.6V, 15mA ≤ I_O ≤ 30mA**	**103**			**103**			**dB**
OPA2211 (per channel)	**A_{OL}**	**(V–) + 0.6V ≤ V_O ≤ (V+) – 0.6V, I_O ≤ 15mA**	**100**						**dB**

errors at room temperature, the minimum A_{OL_DC} from the data sheet should be substituted into Equation 5. Note that in the OPA211 data sheet, "A_{OL_DC}" is written as "A_{OL}."

The first step in this process is to convert A_{OL_DC} from decibels to V/V:

$$A_{OL_DC}\Big|_{V/V} = 10^{\frac{130\ dB}{20}} = 3{,}162{,}278\,\frac{V}{V} \quad (8)$$

$$A_{OL_DC}\Big|_{V/V} = 10^{\frac{114\ dB}{20}} = 501{,}187\,\frac{V}{V} \quad (9)$$

A value for β of 1/200 (the ideal closed-loop gain of 200) can be used in Equation 5 to find the typical DC gain:

$$A_{CL_DC}\Big|_{130\ dB} = \frac{A_{OL_DC}}{1+\beta\times A_{OL_DC}} = \frac{3{,}162{,}278}{1+\frac{1}{200}3{,}162{,}278} = 199.98735 \quad (10)$$

The actual minimum ensured DC gain can be found in the same manner:

$$A_{CL_DC}\Big|_{114\ dB} = \frac{501{,}187}{1+\frac{1}{200}501{,}187} = 199.92022 \quad (11)$$

The DC gain error caused by the open-loop-gain value of the op amp can then be calculated:

$$\varepsilon_{typ} = \frac{A_{CL_DC(ideal)} - A_{CL_DC}}{A_{CL_DC(ideal)}}\times 100 = \frac{200-199.98735}{200}\times 100 = 0.00632\% \quad (12)$$

$$\varepsilon_{max} = \frac{200-199.92022}{200}\times 100 = 0.0399\% \quad (13)$$

The actual DC closed-loop gain of 199.92 has an error of 0.0399% compared to the desired ideal gain of 200.

Over temperature, the OPA211 is characterized to ensure that A_{OL_DC} is higher than 110 dB over the specified temperature range and when loaded with less than 15-mA output current, which is the absolute worst case. For this value, in terms of V/V, 110 dB is equivalent to

$$A_{OL_DC}\Big|_{V/V} = 10^{\frac{110\ dB}{20}} = 316{,}228\,\frac{V}{V}. \quad (14)$$

This number can be substituted into Equation 5 to find the absolute worst-case condition for the DC closed-loop gain:

$$A_{CL_DC}\Big|_{110\ dB} = \frac{316{,}228}{1+\frac{1}{200}316{,}228} = 199.8736 \quad (15)$$

The gain error for this result, 0.063%, represents a slight degradation from the room-temperature case of 0.0399% previously calculated in Equation 13.

DC gain error for inverting configuration

To illustrate the impact of the op amp's finite open-loop gain on the accuracy of DC closed-loop gain in an inverting configuration, another step-by-step example will be presented of calculating the gain error when the op amp is set in an ideal closed-loop gain. This example will use an ideal closed-loop gain of –200 (–α/β = –200), as shown in Figure 3. So that results can be properly compared, the same op amp, OPA211, will be used.

Similar to the non-inverting case, to calculate the DC closed-loop-gain error of the inverting op amp, Equation 3 is first evaluated for zero frequency (f = 0 Hz):

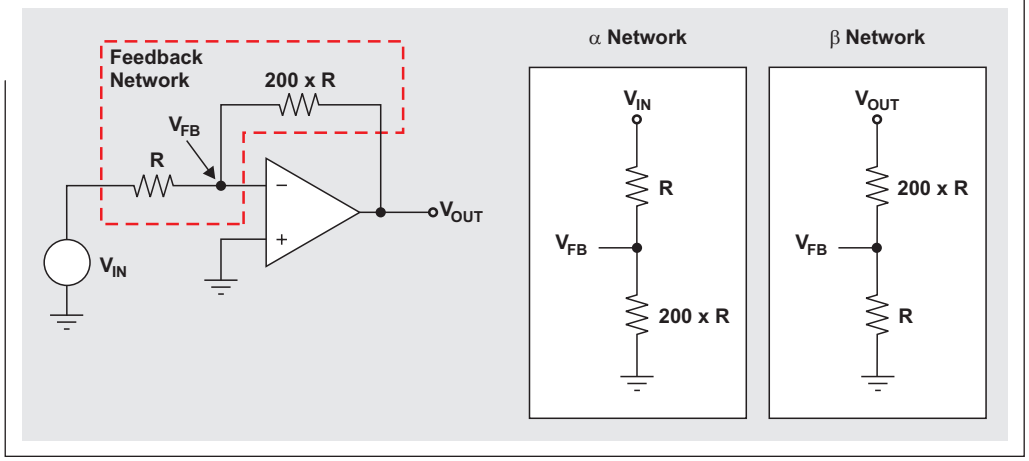
$$A_{CL_DC} = A_{CL}(0\ Hz) = -\alpha\frac{A_{OL_DC}}{1+\beta\times A_{OL_DC}} \quad (16)$$

The negative sign indicates the inverting configuration.

In the case of an ideal op amp with infinite open-loop gain, the DC closed-loop gain of the inverting configuration is reduced to

$$A_{CL_DC(ideal)} = \lim_{A_{OL_DC}\to\infty} -\alpha\frac{A_{OL_DC}}{1+\beta\times A_{OL_DC}} = -\frac{\alpha}{\beta}. \quad (17)$$

Figure 3. Inverting op amp configuration with ideal closed-loop gain of –200



As in the non-inverting configuration, the DC closed-loop gain is entirely determined by the external feedback network.

With the same open-loop-gain specifications of 130 dB (typical) and 114 dB (minimum) at room temperature, and 110 dB (minimum) across the specified temperature range—i.e., the worst case—the same calculations can be done for the inverting configuration as were done for the non-inverting configuration. For an inverting amplifier with an ideal closed-loop gain of –200 ($-\alpha/\beta = -200$), the coefficients $\alpha = 200/201$ and $\beta = 1/201$ can be used for the following three gain calculations.

• Typical DC gain:

$$A_{CL_DC}\Big|_{130\,dB} = -\alpha\frac{A_{OL_DC}}{1+\beta\times A_{OL_DC}} = -\frac{200}{201}\times\frac{3{,}162{,}278}{1+\frac{1}{201}\times 3{,}162{,}278} = -199.98729 \quad (18)$$

• Minimum ensured DC gain at room temperature:

$$A_{CL_DC}\Big|_{114\,dB} = -\frac{200}{201}\times\frac{501{,}187}{1+\frac{1}{201}\times 501{,}187} = -199.9198 \quad (19)$$

• Worst-case DC closed-loop gain over temperature:

$$A_{CL_DC}\Big|_{110\,dB} = -\frac{200}{201}\times\frac{316{,}228}{1+\frac{1}{201}\times 316{,}228} = -199.87296 \quad (20)$$

The DC gain error caused by the variation of the open-loop-gain value of the op amp can then be calculated:

$$\varepsilon_{typ} = \frac{A_{CL_DC(ideal)} - A_{CL_DC}}{A_{CL_DC(ideal)}}\times 100 = \frac{200-199.98729}{200}\times 100 = 0.00636\% \quad (21)$$

$$\varepsilon_{max} = \frac{200-199.9198}{200}\times 100 = 0.0401\% \quad (22)$$

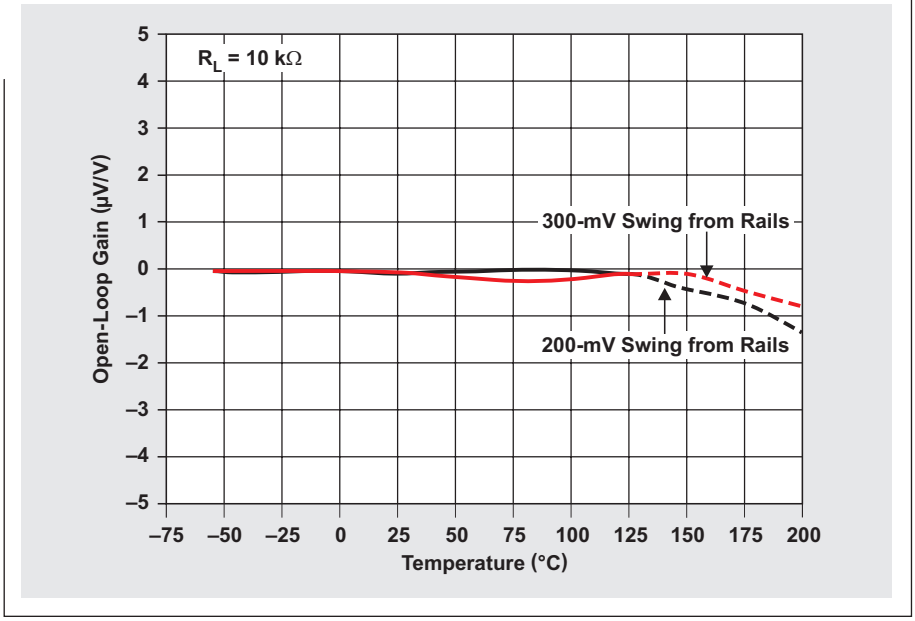
The calculated absolute worst-case condition over temperature for the DC closed-loop gain for the inverting configuration is 0.0635%, compared to 0.0632% for the non-inverting configuration. This example shows that the difference between the non-inverting and inverting configurations is minimal and in many cases can be ignored.

Normalized open-loop gain versus temperature

It should be clear at this point that the DC closed-loop gain is determined by the DC open-loop gain (A_{OL_DC}) of the op amp. Thus, the stability of the DC open-loop gain determines the stability of the DC closed-loop gain. The stability of the open-loop DC gain is determined by many factors, such as the power-supply rejection ratio (PSRR), the temperature, and process variations.

Figure 4 shows the OPA211's normalized DC open-loop gain versus temperature. Note that the changes in open-loop gain are shown in µV/V. As an alternative to representing changes in A_{OL_DC} with decibels as before, A_{OL_DC} can also be represented in terms of µV/V. This representation shows the ratio of the op amp's change in input voltage (error or offset) to the change in its output voltage. In

Figure 4. OPA211's normalized DC open-loop gain versus temperature



other words, the µV/V values have an inverse correlation to the decibel values. As an example, an op amp with an open-loop gain of 199,526 V/V can be written in terms of decibels as

$$A_{OL_DC}\Big|_{V/V} = \frac{V_{OUT}}{V_{ERR}} = 199{,}526\,\frac{V}{V} \quad \textbf{(23)}$$

and

$$A_{OL_DC}\Big|_{dB} = 20\log\frac{V_{OUT}}{V_{ERR}} = 20\log(199{,}526) = 106\text{ dB}. \quad \textbf{(24)}$$

In terms of µV/V, the same gain is written as

$$A_{OL_DC}\Big|_{\mu V/V} = \frac{V_{ERR}}{V_{OUT}} = \frac{1}{199{,}526} = 5.012\,\frac{\mu V}{V}. \quad \textbf{(25)}$$

Figure 4 shows how the OPA211's A_{OL_DC} (in terms of µV/V) may change over temperature. For a device with a given A_{OL_DC} at room temperature (25°C), A_{OL_DC} will typically change less than 0.25 µV/V in the specified temperature range (–40°C to 125°C). For example, if the typical A_{OL_DC} performance is 130 dB, or 0.32 µV/V, at room temperature, then over the specified temperature range, A_{OL_DC} may typically vary between 0.32 µV/V and 0.57 µV/V. To ensure stable operation over temperature, the minimum gain is as follows:

$$A_{OL_DC}\Big|_{V/V} = \frac{V_{ERR}}{V_{OUT}} = \frac{1}{0.57\ \mu V} = 1{,}754{,}386\,\frac{V}{V} \quad \textbf{(26)}$$

$$A_{OL_DC}\Big|_{dB} = 20\log(1{,}754{,}386) = 124.88\text{ dB} \quad \textbf{(27)}$$

This is equivalent to an A_{OL_DC} ranging from 124.88 dB to 130 dB. Keep in mind that these are typical data. It is suggested that, during the circuit-design process, the designer not use typical values but instead use minimum ensured values published by the op amp's manufacturer.

Note that none of the calculations in this article include other factors that also affect A_{OL_DC}, such as the PSRR or the common-mode rejection ratio. The procedure to include these types of errors is similar: Simply add the additional error to the A_{OL_DC} term and recalculate the closed-loop DC gain.

Conclusion

Part 1 of this article series explored general feedback-control-system analysis and synthesis as they apply to first-order transfer functions. The analysis technique was applied to both non-inverting and inverting op amp circuits, resulting in a frequency-domain transfer function for each configuration. Part 2 has shown how to use these two transfer functions and manufacturer data-sheet specifications to analyze the DC gain error of a closed-loop op amp circuit. This analysis also took into consideration the temperature dependency of the open-loop gain as well as its finite value. Part 3 will explore the frequency dependency of the closed-loop gain, which will help designers avoid the common mistake of using DC gain calculations for AC-domain analysis.

References

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Related Web sites

amplifier.ti.com
www.ti.com/sc/device/OPA211

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