Operational amplifier gain stability, Part 3: AC gain-error analysis

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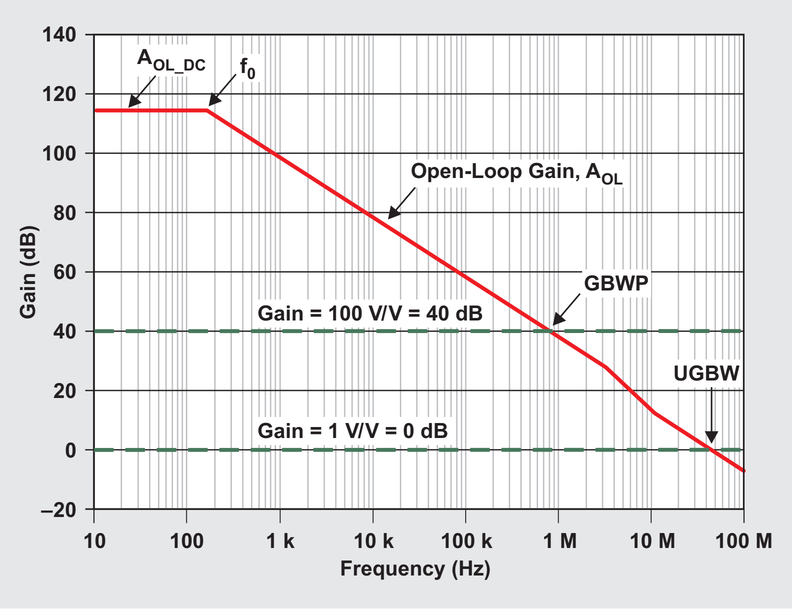
Introduction

The goal of this three-part series of articles is to provide readers with an in-depth understanding of gain accuracy in closed-loop circuits with the most typical operational amplifier (op amp) configurations: non-inverting and inverting. Often, the effects of various op amp parameters on the accuracy of the circuit's closed-loop gain are overlooked and cause an unexpected gain error both in the DC and the AC domains.

In Part 1 (Reference 1), two separate equations were derived for calculating the transfer functions of non-inverting and inverting op amps. Part 2 (Reference 2) showed how to use these two transfer functions and manufacturer data-sheet specifications to analyze the DC gain error of a closed-loop op amp circuit. The same article also discussed how open-loop gain dependency on temperature affects the op amp closed-loop gain error across its specified operating temperature range.

This final article, Part 3, explores the frequency dependency of the closed-loop gain, which will help designers avoid the common mistake of using DC gain calculations for AC-domain analysis.

Figure 1. OPA211 open-loop gain versus frequency



The significance of the gain-bandwidth product

This section will review the concept of the op amp gain-bandwidth product (GBWP), G × BW. The GBWP is a parameter that is needed before the AC closed-loop gain can be calculated. First, GBWP (or GBP, as it is sometimes referred to) is needed to calculate the op amp closed-loop cutoff frequency. GBWP is also needed to calculate the frequency of the dominant pole, f_0, of the op amp open-loop response. At frequencies below f_0, the DC gain-error calculation in Part 2 is valid because the open-loop gain of the op amp is constant; this gain is equal to the A_{OL_DC} (see References 1 and 2). However, beyond a frequency of f_0, the AC calculation must be used, as will be discussed in the following section.

In general, if an op amp has a straight, –20-dB/decade, open-loop-gain rolloff, it has a constant GBWP. For a chosen closed-loop gain, the cutoff frequency at which the closed-loop gain starts to roll off can be calculated by dividing the GBWP by the desired closed-loop gain. Note that in practice the resulting –3-dB point of the closed-loop response may not be exactly equal to the calculated rolloff point due to gain peaking and other non-ideal factors.

Figure 1 shows the simplified open-loop gain versus the frequency response for the Texas Instruments (TI) OPA211. In the product data sheet, the GBWP is specified for two different gains: 1 (GBWP = 45 MHz) and 100 (GBWP = 80 MHz). The reason for the two different gain specifications is that the OPA211's open-loop gain response has an additional pole-zero pair in the frequency region from about 4 to 20 MHz. This is a special case that is contrary to the earlier statement that op amps with straight –20-dB/decade rolloffs will have only one GBWP. For this reason, the GBWP of 80 MHz should be used for calculating the cutoff frequencies for op amps that have a closed-loop gain of 100 or higher, and the GBWP of 45 MHz should be used for op amps with a closed-loop gain of 2 or lower. If a more precise calculation is needed in the frequency region above 4 MHz, using SPICE simulation is suggested.

Using the specified GBWP lets the designer calculate cutoff frequencies for different closed-loop gains. When the op amp is in the unity-gain configuration (where the closed-loop gain is 1), the cutoff frequency is 45 MHz (45 MHz/1), which is also known as the unity-gain bandwidth (UGBW) of the op amp. If the op amp has a closed-loop gain of 100, the cutoff frequency is 800 kHz (80 MHz/100).

To calculate the OPA211's dominant-pole frequency (f_0), the GBWP of 80 MHz will be used. Again, 80 MHz is valid for a closed-loop gain of 100 or higher, up to the value of A_{OL_DC}. A value of 114 dB, which is the minimum ensured DC open-loop gain for the OPA211 at room temperature, will be used for A_{OL_DC}. Substituting all these parameters into Equation 1 yields

$$f_0 = \frac{GBWP}{A_{OL\_DC}} = \frac{80\ \text{MHz}}{10^{\frac{114\ \text{dB}}{20}}} = 159.62\ \text{Hz}. \qquad \textbf{(1)}$$

This result will be used in the following section to calculate the AC closed-loop gain.

Calculating the AC closed-loop gain

In Part 1, the closed-loop transfer function of the non-inverting op amp configuration in the frequency domain was calculated. Specifically, the transfer function was derived with the assumption that the op amp had a first-order open-loop response. For calculating gain error, the magnitude response is of interest. For convenience, the result is repeated in the following equation:

$$\left|A_{CL}(f)\right|_{dB} = 20\log\frac{\dfrac{A_{OL\_DC}}{1+\beta\times A_{OL\_DC}}}{\sqrt{1+\dfrac{f^2}{f_0^2}\times\dfrac{1}{(1+\beta\times A_{OL\_DC})^2}}}, \qquad \textbf{(2)}$$

where β is defined as

$$\beta = \frac{V_{FB}}{V_{OUT}} = \frac{R_I}{R_I+R_F}. \qquad \textbf{(3)}$$

Also derived in the same article was the equation for calculating the magnitude of the inverting configuration's closed-loop gain. The result is repeated in Equation 4:

$$\left|A_{CL}(f)\right|_{dB} = 20\log\frac{\alpha\dfrac{A_{OL\_DC}}{1+\beta\times A_{OL\_DC}}}{\sqrt{1+\dfrac{f^2}{f_0^2}\times\dfrac{1}{(1+\beta\times A_{OL\_DC})^2}}} \qquad \textbf{(4)}$$

Equation 4 uses the same variable β defined by Equation 3. Additionally, the variable α is defined by Equation 5:

$$\alpha = \frac{V_{FB}}{V_{IN}} = \frac{R_F}{R_I+R_F} \qquad \textbf{(5)}$$

At this point, the closed-loop gain for non-inverting and inverting amplifiers is represented by Equations 2 and 4, respectively. These equations calculate the magnitude of the transfer functions and will be used for subsequent analysis.

In Part 2, the DC closed-loop transfer function of the non-inverting op amp configuration was calculated. Again, the transfer function was derived with the assumption that the op amp had a first-order open-loop response. The DC closed-loop gain of the non-inverting and inverting amplifiers can be derived by setting f equal to 0 in Equations 2 and 4, which yields the following two equations:

$$A_{CL\_DC} = \frac{A_{OL\_DC}}{1+\beta\times A_{OL\_DC}} \qquad \textbf{(6)}$$

$$A_{CL\_DC} = -\alpha\frac{A_{OL\_DC}}{1+\beta\times A_{OL\_DC}} \qquad \textbf{(7)}$$

The DC closed-loop gain was derived in slightly different ways in other published articles (References 3 to 8); however, the results agree with this analysis. Unfortunately, in these same articles, the expressions for the AC closed-loop gain were derived by simply replacing A_{OL_DC} with $A_{OL}(f)$ in Equations 6 and 7, which represent the simple transfer functions. The results are shown in Equations 8 and 9:

$$A_{CL}(f) = \frac{A_{OL}(f)}{1+\beta\times A_{OL}(f)} \qquad \textbf{(8)}$$

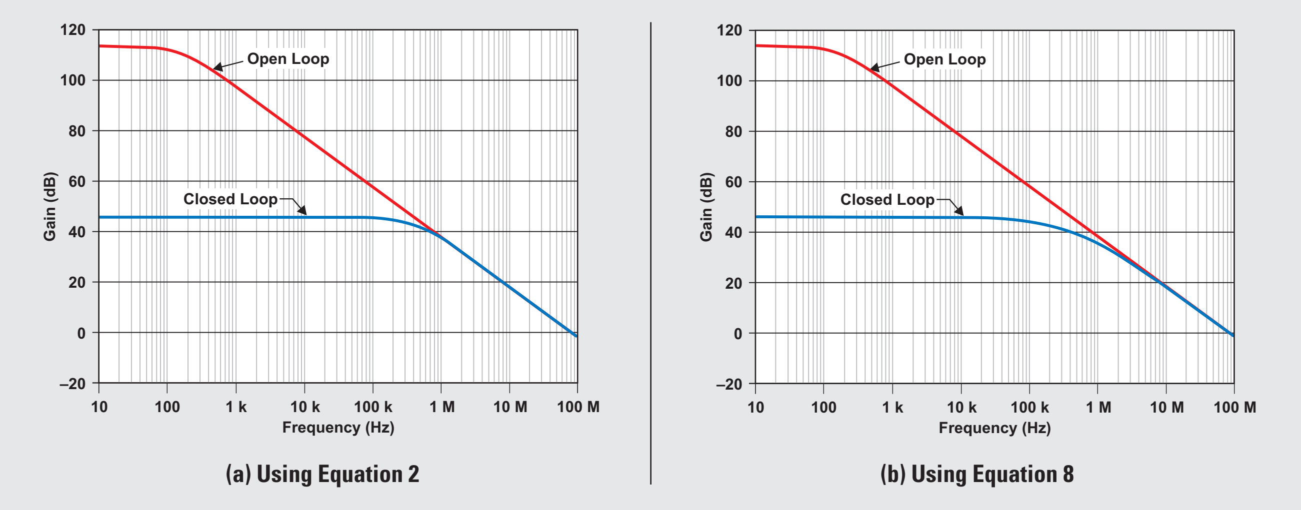
$$A_{CL}(f) = -\alpha\frac{A_{OL}(f)}{1+\beta\times A_{OL}(f)} \qquad \textbf{(9)}$$

In these two equations, assuming a first-order system, $A_{OL}(f)$ is defined as

$$A_{OL}(f)\big|_{dB} = A_{OL\_DC}\big|_{dB} - 20\log\sqrt{1+\frac{f^2}{f_0^2}}. \qquad \textbf{(10)}$$

However, this is not the correct way to calculate AC closed-loop gain. Instead, Equations 2 and 4, which are the magnitude expressions of the closed-loop transfer function, should be used. Equation 2 should be used instead of Equation 8 for a non-inverting configuration, and Equation 4 should be used instead of Equation 9 for an inverting configuration. The next two sections will show the difference in results when the correct and incorrect equations are used to calculate the gain.

Figure 2. Closed-loop response of OPA211 in non-inverting configuration (G = 200 V/V)



(a) Using Equation 2

(b) Using Equation 8

Table 1. Closed-loop gain of OPA211 in non-inverting configuration (G = 200 V/V or 46 dB)

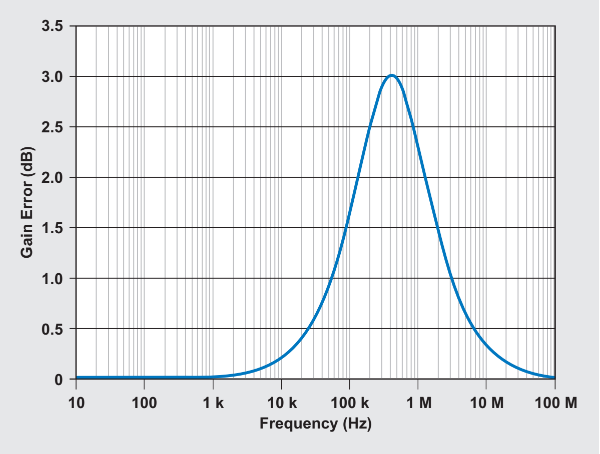
FREQUENCY (kHz)	CLOSED-LOOP GAIN CALCULATED WITH EQUATION 8		CLOSED-LOOP GAIN CALCULATED WITH EQUATION 2		CLOSED-LOOP GAIN ERROR RESULTING FROM EQUATION 8	
	(V/V)	(dB)	(V/V)	(dB)	(%)	(dB)
10	195.121	45.806	199.86	46.014	2.37	0.208
30	186.046	45.392	199.361	45.993	6.679	0.6
60	173.913	44.807	197.71	45.921	12.036	1.114
100	160	44.082	193.956	45.754	17.507	1.672
300	114.286	41.16	159.959	44.08	28.553	2.92
600	80	38.062	110.926	40.901	27.88	2.839
1000	57.143	35.139	74.274	37.417	23.065	2.278

AC gain error for non-inverting configuration

As just stated, there is a tendency for system designers to substitute Equation 10 into Equation 8 to calculate AC gain for a non-inverting configuration. Figure 2 shows the difference in the OPA211's closed-loop response when that method is used versus using Equation 2. In this example, the closed-loop gain is set to 200 V/V ($\beta = 1/200$). From Figure 2 it is evident that the difference between using the two equations is primarily in the region of a decade before and after the theoretical intersection between the open-loop and closed-loop curves (that is, the cutoff frequency).

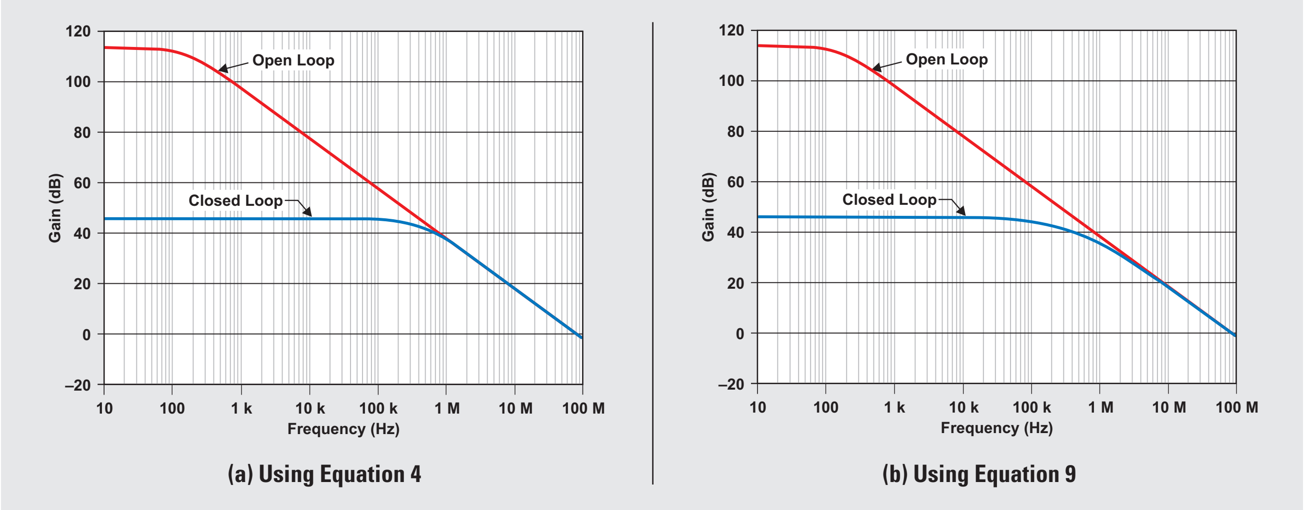
From the previous discussion of the GBWP, it is expected that the OPA211 with a gain of 200 V/V will have a cutoff frequency of 400 kHz (80 MHz/200). Table 1 shows the values in Figure 2 in tabular form for a few selected frequencies. For the frequencies of 10 kHz and 100 kHz, the table shows that there is quite a bit of difference in the frequency responses. The closed-loop gain calculated with Equation 8 drops from about 195 V/V to 160 V/V, compared to a drop of about 199 V/V to 194 V/V with Equation 2. The biggest difference occurs at the cutoff frequency of 400 kHz, where the error is 29%, or 3 dB. These differences, which can be considered as gain error, are plotted in Figure 3.

Figure 3. OPA211 closed-loop gain error resulting from Equation 8



The foregoing analysis shows that a proper understanding of gain error is extremely important in selecting proper components. If a design requires that the flatness of the closed-loop gain be kept within a specified margin, using

Figure 4. Closed-loop response of OPA211 in inverting configuration (G = –200 V/V)



(a) Using Equation 4 **(b) Using Equation 9**

Table 2. Closed-loop gain of OPA211 in inverting configuration (G = –200 V/V or 46 dB)

FREQUENCY (kHz)	CLOSED-LOOP GAIN CALCULATED WITH EQUATION 9		CLOSED-LOOP GAIN CALCULATED WITH EQUATION 4		CLOSED-LOOP GAIN ERROR RESULTING FROM EQUATION 9	
	(V/V)	(dB)	(V/V)	(dB)	(%)	(dB)
10	195.098	45.805	199.857	46.014	2.381	0.209
30	185.981	45.389	199.355	45.993	6.708	0.603
60	173.8	44.801	197.688	45.92	12.084	1.119
100	159.84	44.074	193.898	45.751	17.565	1.678
300	114.041	41.141	159.671	44.065	28.577	2.923
600	79.761	38.036	110.543	40.871	27.847	2.835
1000	56.94	35.108	73.955	37.379	23.008	2.271

Equation 8 will lead the designer to select an op amp with a UGBW 10 times higher than what is needed.

AC gain error for inverting configuration

Similar to the non-inverting configuration, most system designers will use Equations 9 and 10 to calculate the AC gain for an inverting configuration. The difference in the resulting closed-loop gains when Equations 4 and 9 are used is shown in Figure 4. In this example, the op amp is set to an inverting gain of –200 V/V (β = 1/201, α = 200/201). From Figure 4 it can be seen that once again the most significant difference in the results is in the region about a decade before and after the cutoff frequency.

Table 2 shows the values in Figure 4 in tabular form for a few selected frequencies. For the frequencies of 10 kHz and 100 kHz, Table 2 shows the same differences in frequency response as for the non-inverting configuration. The closed-loop gain calculated with Equation 9 drops from about 195 V/V to 160 V/V, compared to a drop of about 199 V/V to 194 V/V with Equation 4. Again, the biggest difference occurs at the cutoff frequency of 400 kHz, where the error is 29%, or 3 dB. These differences, which can be considered as gain error, are plotted in Figure 5 and lead to a conclusion similar to that for the non-inverting configuration: If a design requires that the flatness of the closed-loop gain be kept within a specified margin, using Equation 9 will lead the designer to select an op amp with a UGBW 10 times higher than what is needed.

Figure 5. OPA211 closed-loop gain error resulting from Equation 9

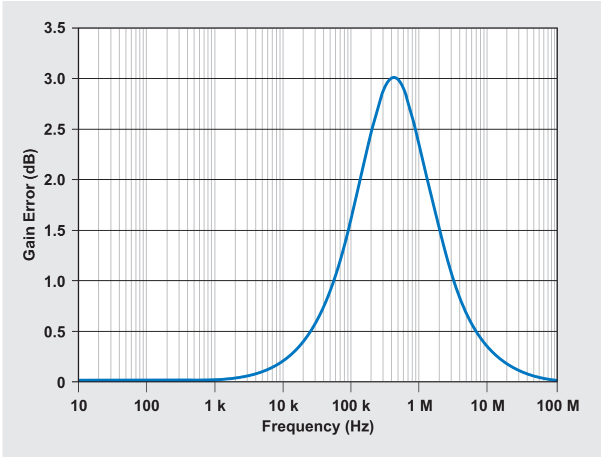


Table 3. Calculated and SPICE-simulation values for AC closed-loop gain

FREQUENCY (kHz)	CLOSED-LOOP GAIN FOR NON-INVERTING CONFIGURATION (V/V)		CLOSED-LOOP GAIN FOR INVERTING CONFIGURATION (V/V)	
	FROM EQUATION 2	FROM SPICE SIMULATION	FROM EQUATION 4	FROM SPICE SIMULATION
10	199.86	199.91	199.86	199.91
30	199.36	199.43	199.36	199.42
60	197.71	197.85	197.69	197.82
100	193.96	194.24	193.89	194.18
300	159.96	161.18	159.67	160.89
600	110.93	112.53	110.54	112.12
1000	74.27	75.5	73.96	75.18

Comparison to SPICE simulation

To verify the validity of Equations 2 and 4 for calculating the AC closed-loop gain in non-inverting and inverting configurations, the results were compared to those of a TINA-TI™ SPICE simulation. For this analysis, the OPA211 macromodel was used. This simulation model can be downloaded at:

http://focus.ti.com/docs/prod/folders/print/opa211.html#toolssoftware

Table 3 shows that the calculated results from Equations 2 and 4 closely match the results from the SPICE simulation, confirming that Equations 2 and 4 are indeed the correct equations to use to calculate the AC closed-loop gain. The slight discrepancies between the calculated and simulated results can be attributed to the fact that the SPICE simulation included non-ideal op amp factors (such as input bias currents, etc.) that were ignored in this simplified analysis.

Conclusion

Part 1 of this article series explored general feedback-control-system analysis and synthesis as they apply to first-order transfer functions. The analysis technique was applied to both non-inverting and inverting op amp circuits, resulting in a frequency-domain transfer function for each configuration.

Part 2 showed how to use these two transfer functions and manufacturer data-sheet specifications to analyze the DC gain error of a closed-loop op amp circuit. This analysis also took into consideration the temperature dependency of the open-loop gain as well as its finite value.

Part 3 of this article series has explored how to calculate the closed-loop gain error for AC input signals. Instead of using the magnitude equations, system designers have a tendency to use the simple transfer-function equations. As has been shown, using these equations will lead to incorrect results, specifically in the vicinity of the circuit's cutoff frequency, where the error will be more significant. By using the magnitude equations to calculate the closed-loop gain, system designers should be able to choose a more appropriate op amp that will meet the design requirements.

References

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