



Stability - 2
TIPL 1332
TI Precision Labs – Op Amps

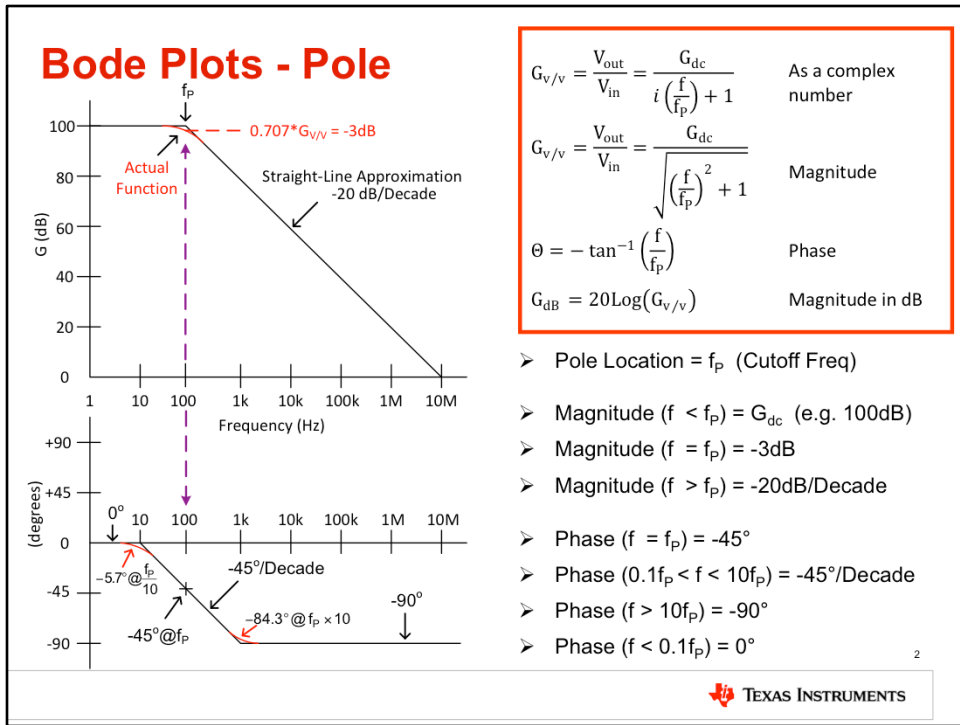
Presented by Collin Wells
Prepared by Collin Wells, Art Kay, Ian Williams, and Tim Green

Prerequisites: Op Amp Bandwidth 1 – 3
(TIPL1221 – TIPL1223)

 TEXAS INSTRUMENTS

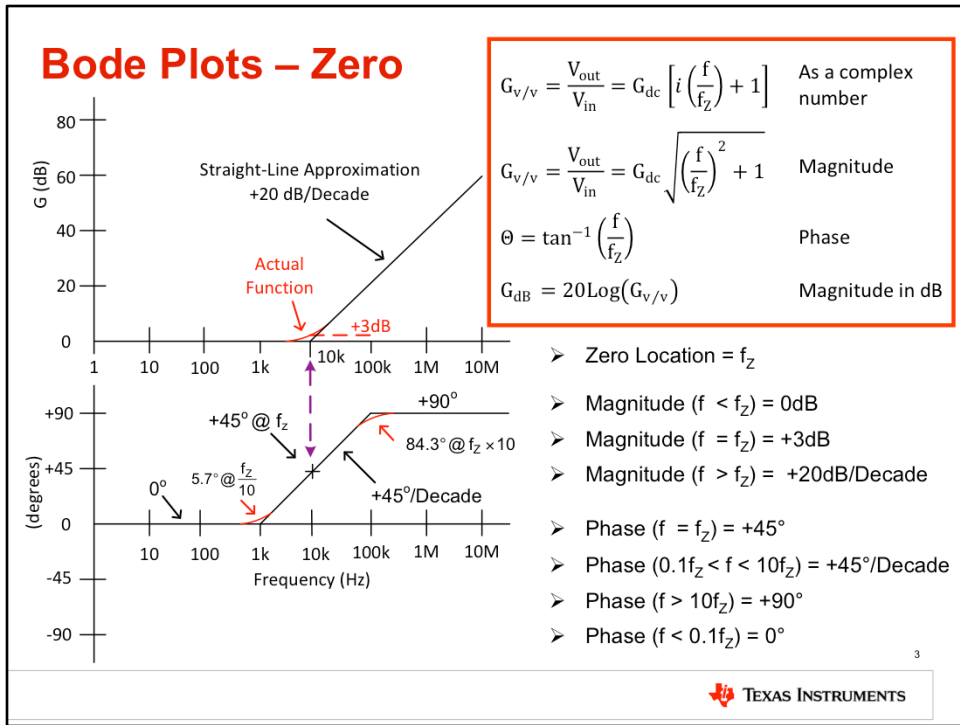
Hello, and welcome to part two of the TI Precision Labs on op amp stability. In the first video we discussed the types of issues that op amp stability can cause in production systems as well as how to identify issues in the lab.

This video will provide a brief review of Bode plots and basic stability theory using phase margin and rate of closure analysis. It is important to thoroughly understand these concepts before proceeding with the video series. Please be sure you've completed the lectures and problem sections for Op-Amp Bandwidth one through three before proceeding.



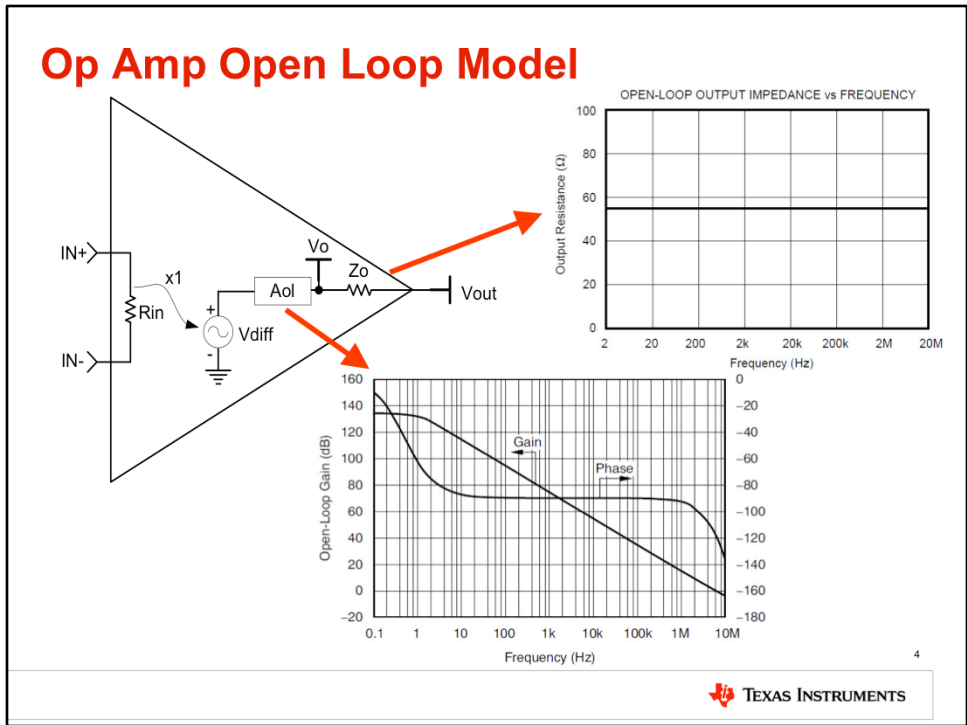
This slide is from op amp bandwidth video number 1. It illustrates the equations for a pole and its associated magnitude and phase response on Bode plots. A pole causes a negative 20 dB/decade decrease in the slope of the magnitude response after the pole frequency, f_p . The pole also causes a negative 90 degree phase shift in the phase response beginning roughly a decade before f_p and ending roughly a decade afterwards.

At f_p the magnitude response will have decreased by negative 3 dB, and the phase will have shifted by negative 45 degrees. While the pole results in a total phase shift of 90 degrees over about 2.5 decades, the phase shift is equal to negative 5.7 degrees a decade before f_p and negative 84.3 degrees a decade after f_p .



This slide, also from op-amp bandwidth video 1, illustrates the equations for a zero and its associated magnitude and phase response on Bode plots. A zero causes a positive 20dB/decade increase in the slope of the magnitude response after the zero frequency, f_z . The zero also causes a positive 90 degree phase shift in the phase response beginning roughly a decade before f_z and ending roughly a decade afterwards.

At f_z the magnitude response has increased by plus 3 dB and the phase has shifted by positive 45 degrees. While the zero results in a total phase shift of positive 90 degrees over about 2.5 decades, the phase shift is equal to positive 5.7 degrees a decade before f_z and positive 84.3 degrees a decade after f_z .

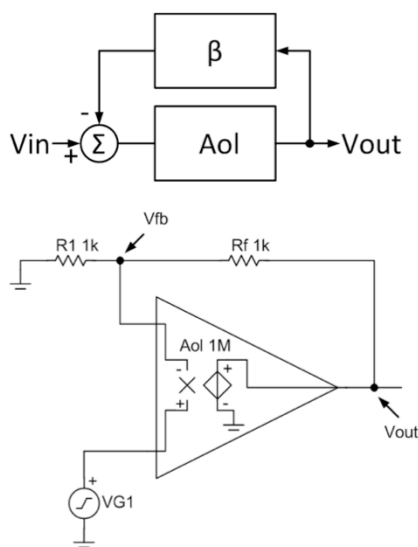


It is helpful to use an intuitive model for the op amp when performing ac stability analysis because of the complexity of modern op amps. In this simplified “stability” model, the differential input voltage applied to the inputs is passed to the amplifier output stage where it passes through the amplifier open-loop gain, followed by the open-loop output impedance before it reaches the output terminal.

The open-loop gain, or A_{ol} , of an op amp represents the maximum gain that can be applied over frequency to the differential voltage applied between the inputs of the device. A_{ol} for an ideal amplifier is infinite and is not limited by frequency. Modern op amps can have open loop gains in excess of 1 million volts per volt, or 120dB at low frequencies and unity-gain bandwidths from 10’s of kHz up to several GHz.

The open-loop output impedance, Z_o , is a measure of the impedance of the open-loop output stage of the op amp. Z_o should not be confused with the amplifier’s closed-loop output impedance, Z_{out} , which depends on Z_o , A_{ol} , and the circuit configuration. To keep the stability analysis focused on the basic concepts for this series, the behavior of the Z_o will be viewed as a resistor over all frequencies of interest. In truth Z_o can vary widely over frequency for newer rail-to-rail output stages making stability analysis more difficult. Complex output impedance will be discussed in the advanced section at the end of this series after a firm understanding of analysis with resistive output impedance has been developed.

Op Amp Closed Loop Model



A_{ol} = Open loop Gain

$$\beta = \text{Feedback Factor} = \frac{V_{fb}}{V_{out}} = \frac{R_1}{R_1 + R_f}$$

$$A_{cl} = \text{Closed Loop Gain} = \frac{A_{ol}}{1 + A_{ol}\beta}$$

$A_{ol}\beta$ = Loop Gain

$$A_{cl} = \lim_{A_{ol}\beta \rightarrow \infty} \left(\frac{A_{ol}}{1 + A_{ol}\beta} \right) = \frac{1}{\beta} = 1 + \frac{R_f}{R_1}$$

TEXAS INSTRUMENTS

To control the large open-loop gain of modern amplifiers, negative feedback is required between the output of the amplifier and the inverting input. This is referred to as “closing the loop.” In this circuit, the loop is closed with R_f and R_1 which create a voltage divider, and therefore an attenuation, between the output and the inverting input. The ratio of the resistors determines the amount of the output that is fed back to the input which is defined as the feedback factor, or Beta, of the circuit.

Closing the loop results in a closed-loop gain, A_{cl} , that is equal to A_{ol} divided by 1 plus A_{ol} multiplied by Beta. A_{ol} multiplied by Beta is referred to as Loop-gain. When the loop-gain is large, the closed-loop gain formula can be simplified to equal $1/\text{Beta}$. In this example $1/\text{Beta}$ equals $1+R_f/R_1$, which can be recognized as the gain of a non-inverting amplifier circuit.

Closed-loop gain through negative feedback is a fundamental concept in amplifier circuit design and should be thoroughly understood. Let’s review it again quickly. The amplifier will adjust its output to equalize the two inputs establishing the virtual short between them. Therefore an attenuation from the output to the input, set by Beta, forces the output to be larger than the input by the inverse of Beta. This is how the ratio of the feedback resistors sets the closed-loop gain of the circuit. .

When is an Amplifier Unstable?

$$A_{CL} = A_{OL} / (1 + A_{OL}\beta)$$

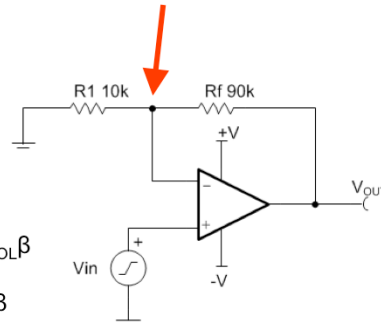
- A circuit is unstable when $A_{OL}\beta = -1$
- $A_{OL}\beta = -1$ sets the denominator of $A_{CL} = 0$
- $A_{OL}\beta = -1$ when $A_{OL}\beta(\text{dB}) = 0\text{dB}$ and phase shift($A_{OL}\beta$) = 180°
 - Phase shift is relative to the DC phase

Phase Margin (PM)

How close the system is to a 180° phase shift in $A_{OL}\beta$

- $\text{PM} = \text{Phase}(A_{OL}\beta)$ when $\text{Gain}(A_{OL}\beta) = 0\text{dB}$
- Ex: 10° phase margin = 170° phase shift in $A_{OL}\beta$

$A_{OL}\beta = -1$ when the phase at V_{FB} has shifted 180° relative to V_{in}



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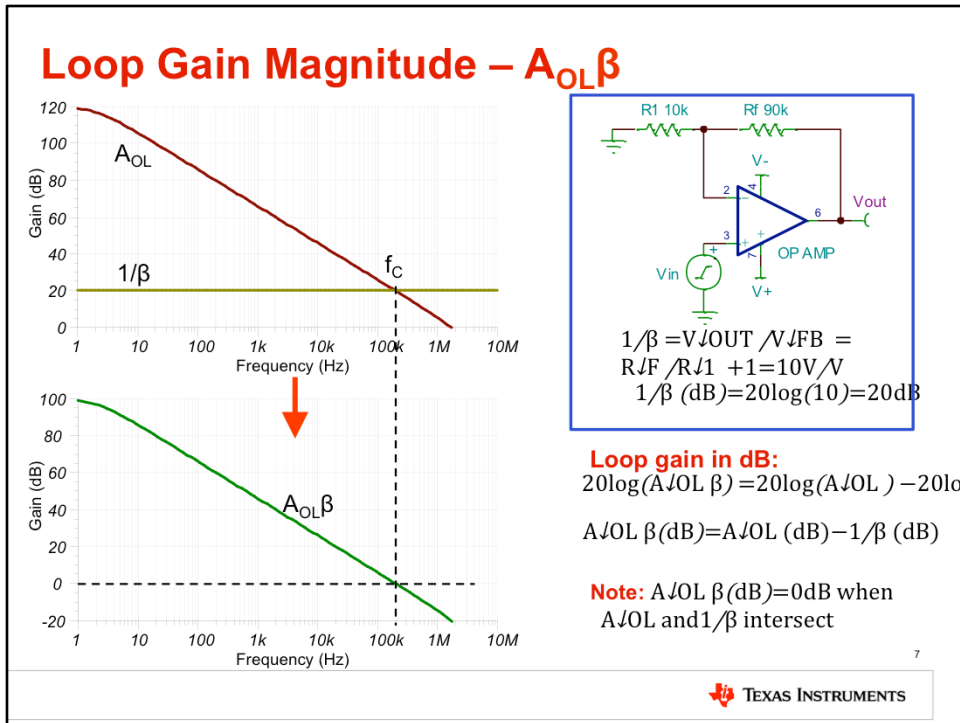
Let's now move on and define the conditions for stability using mathematical and graphical methods.

First, we must define when an amplifier is unstable. Looking back at the op amp closed-loop gain equation, we remember that $A_{cl} = A_{ol} / (1 + A_{ol}B)$. Taking a closer look, we can see that if $A_{ol}B$, or the loop gain, equals -1 , we get zero in the denominator and therefore A_{cl} becomes undefined. This is the mathematical definition of instability.

How can this happen in a real circuit?

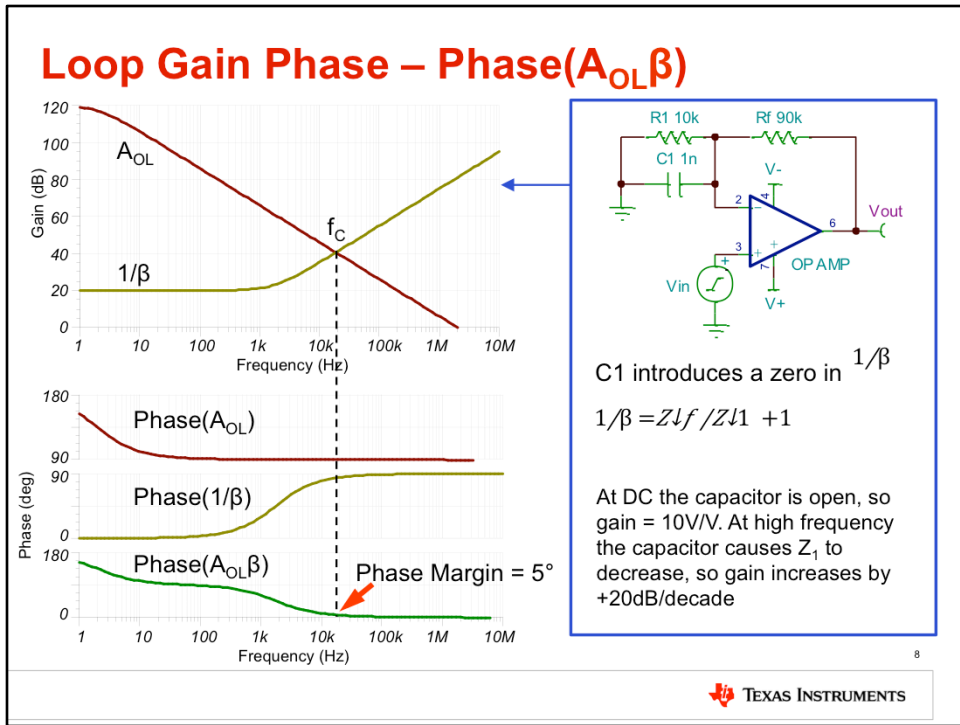
Well, at some point in frequency $A_{ol}B$ will equal 0dB , which is equal to $1V/V$. If enough delay is introduced into the feedback path, the phase in the feedback network will shift 180 degrees relative to V_{in} . A 180 degree phase shift is equivalent to an inversion of the input, or -1 . Therefore, when the gain of $A_{ol}B = 0\text{dB}$ and the phase has shifted by 180 degrees, the result is $A_{ol}B = -1$.

The term "Phase Margin" is used to define how close a circuit is to this condition. Phase margin is simply the phase of $A_{ol}B$ at the frequency where $A_{ol}B = 0\text{dB}$. For example, 10 degrees of phase margin means that $A_{ol}B$ has shifted by 170 degrees at the point where $A_{ol}B = 0\text{dB}$.



First we can consider the loop gain magnitude using a Bode plot. Using the same circuit as before, we have a gain of $10V/V$, or 20dB , so $1/\beta$ is a constant 20dB over frequency. The circuit's A_{OL} is also shown. To find the magnitude of $A_{OL}\beta$, we can simply subtract $1/\beta$ from A_{OL} . This might not seem intuitive, but the mathematical relationship shown on the right side of the slide proves this using the properties of logarithms.

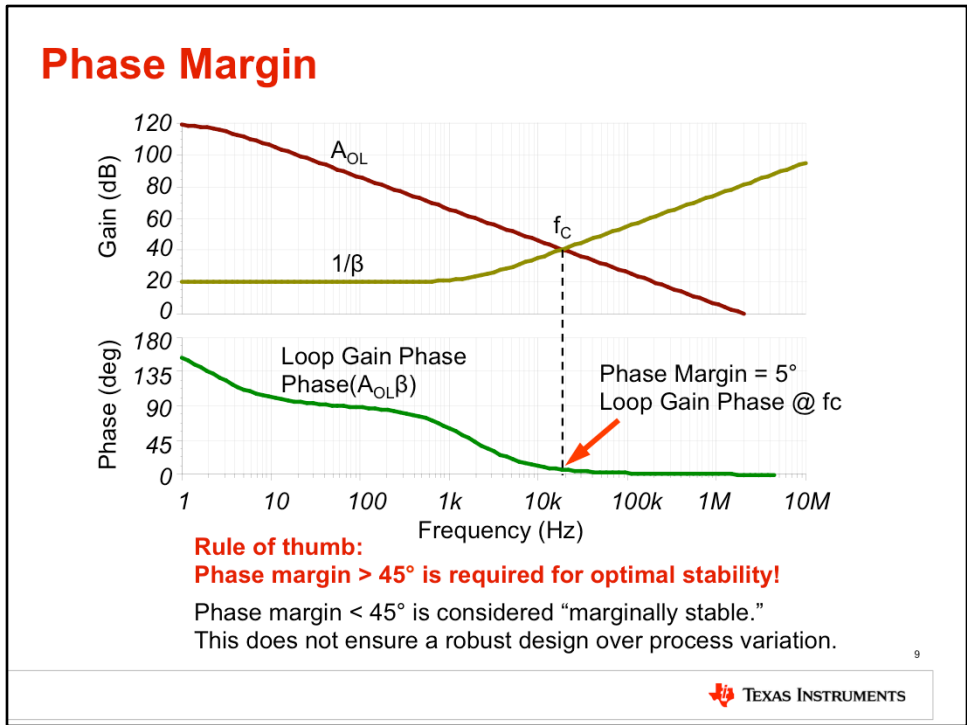
Remember in the last slide we stated that the phase margin is the loop gain phase at the frequency where $A_{OL}\beta = 0$. This frequency is called " f_c " and defines where the loop is closed. This is also the frequency where A_{OL} and $1/\beta$ intersect, which makes sense since the difference of two equal values is zero.



To measure the phase margin, we need to know the loop gain phase, or phase of $A_{OL}\beta$, over frequency. Using the same log properties as before, we can simply subtract the phase of $1/\beta$ from the phase of A_{OL} to get the phase of $A_{OL}\beta$.

In this example, a capacitor was added to the feedback network of the op amp circuit. At DC the capacitor is open, so the closed-loop gain is 10V/V like the previous circuit. At some higher frequency, the capacitor causes the impedance of the combination of R_1 and C_1 to decrease, so the gain of the circuit increases by +20dB/decade. This can be seen from the zero in the $1/\beta$ plot.

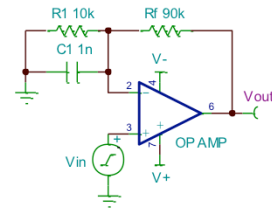
Looking at the phase, the 90° increase in the phase of $1/\beta$ creates a 90° decrease in the phase of $A_{OL}\beta$, so phase margin becomes very low at only 5° .



Now that we know how to observe phase margin, let's review what it's actually telling us. Remember that what we want is to avoid the condition where the loop gain, $A_{OL}\beta$, equals -1. That means we have a phase shift of 180 degrees at f_c , or 0 degrees of phase margin. For optimal stability, we use a rule of thumb which states that a phase margin of 45 degrees or higher is required. While a circuit may work with phase margin less than 45 degrees, it is considered to be only marginally stable and will still show significant overshoot and ringing. Also keep in mind that over the lifetime of a product, devices will have different characteristics due to process variation, temperature, component value tolerances, and other fluctuations. Therefore, for a robust design you should really meet the 45 degrees of phase margin minimum requirement.

Instead of having to directly measure the phase margin of every circuit to verify stability, another analysis method exists which is simpler and can actually give more information about what causes the stability problem in a circuit.

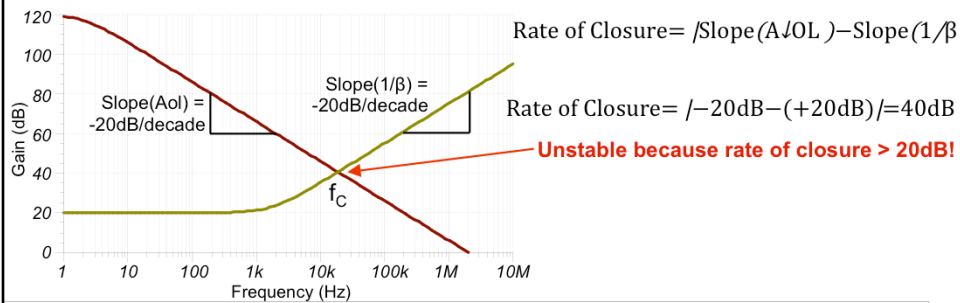
Rate of Closure – Unstable Example



$$1/\beta = V_{OUT} / V_{FB} = 10(f/f_c + 1)$$

$1/\beta(\text{dB}) = 20\text{dB}$ at DC, then increases by $+20\text{dB/decade}$ after the zero frequency

Rule of thumb:
Rate of closure = 20dB is required for optimal stability!



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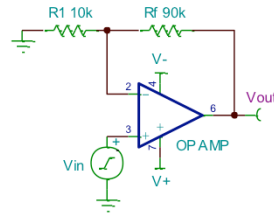
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This method is called rate of closure analysis. To use this method, we only need to consider the magnitude plots of A_{OL} and $1/\beta$. These plots have well-defined slopes due to the poles and zeros in their transfer functions. By analyzing the rate of closure of A_{OL} and $1/\beta$ at f_c , the point where they intersect, we can quickly determine the stability of a circuit. The rule of thumb for this method is that the rate of closure at f_c must equal 20dB for optimal stability.

Let's use our same circuit example from earlier with a capacitor on the op amp inverting input. That capacitor causes a zero in $1/\beta$, which makes $1/\beta$ increase with a slope of 20dB/decade. The A_{OL} curve of the op amp decreases at 20dB/decade due to the op amp's dominant pole. When they intersect at f_c , the rate of closure is the absolute value of the slope of A_{OL} minus the slope of $1/\beta$, which works out to be 40dB. Since the rate of closure is greater than 20dB, we can conclude that the circuit is unstable, matching the poor phase margin measured previously.

Besides being quick and easy to check, the rate of closure method provides additional information into the cause of the circuit instability. In this example, the slope of A_{OL} only shows the effect of the op amp dominant pole as we expect. However, the rise in $1/\beta$ indicates a zero in the feedback network, so we can then take steps to compensate for it. Measuring the phase margin does not provide this level of insight into the cause of the stability issue, only whether or not there is an issue.

Rate of Closure – Stable Example

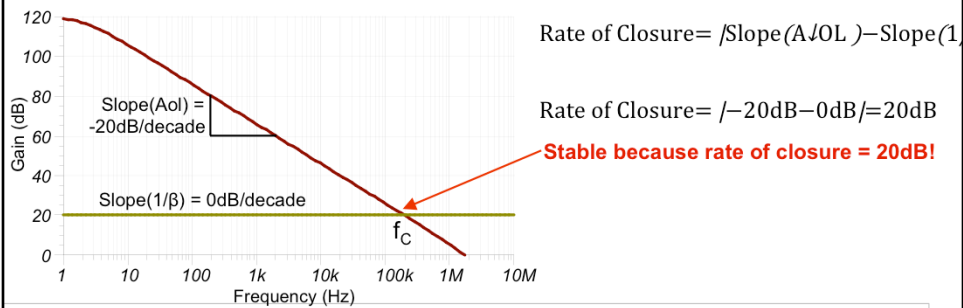


$$\frac{1}{\beta} = \frac{V_{OUT}}{V_{FB}} = \frac{R_f}{R_1} + 1 = 10V/V$$

$$\frac{1}{\beta} \text{ (dB)} = 20 \log(10) = 20 \text{ dB}$$

Rule of thumb:

Rate of closure = 20dB is required for optimal stability!

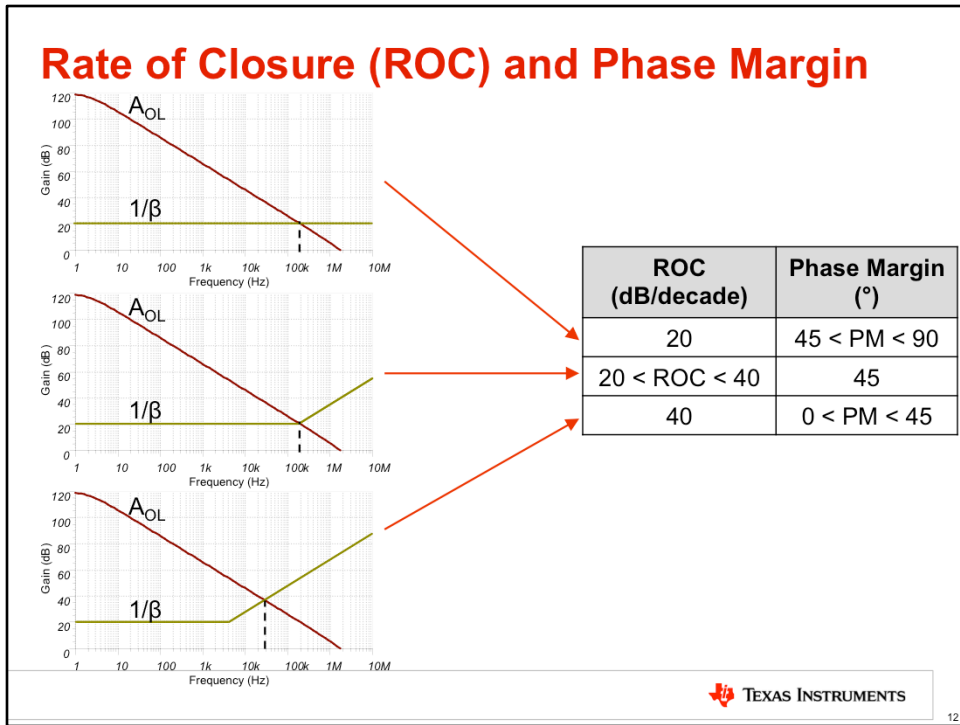


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We can also analyze the standard non-inverting amplifier circuit from earlier in the lecture using the rate of closure method.

In this case, $1/\beta$ is flat and doesn't contain the zero shown in the previous example. A_{ol} still decreases with a slope of -20dB/decade as before and therefore, the rate of closure is now the absolute value of $-20\text{dB} - 0\text{dB}$, or 20dB . We can conclude that this circuit is stable.



As shown in the previous slides, rate of closure and phase margin are directly related to each other allowing us to predict one value based on the other. This slide gives three different examples of rate of closure and their corresponding phase margins.

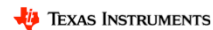
In the first case, we have a rate of closure of 20dB/decade, so the circuit is stable and we have between 45 and 90 degrees of phase margin. This is the best case for circuit design.

In the second case, we have a zero in $1/\beta$ right at f_c . At f_c the rate of closure is beginning to change and will be somewhere between 20 and 40dB per decade. This case corresponds to roughly 45 degrees of phase margin. Remember that a zero causes a total phase shift of 90 degrees with 45 degrees of phase shift right at the zero frequency. Therefore the $A_{OL}\beta$ phase will have 90 degrees of phase shift from the A_{OL} dominant pole and 45 degrees of phase shift from the $1/\beta$ zero at f_c , leaving 45 degrees of phase margin.

In the final case, there is a zero in $1/\beta$ well before f_c , so the rate of closure is 40dB/decade. This results between 0 and 45 degrees of phase margin corresponding to an unstable circuit.

**Thanks for your time!
Please try the quiz.**

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In summary, this video discussed several fundamental concepts in op amp stability theory including a Bode plot review, phase margin and rate-of-closure analysis.

Stay tuned for the next videos which will cover the theory for simulating op-amp stability in SPICE.

Thank you for time! Please try the quiz to check your understanding of this video's content.

Stability 2

Multiple Choice Quiz

TI Precision Labs – Op Amps



Quiz: Stability 2

1. A pole in a transfer function results in:

- a. A -20dB/decade decrease in gain and a -90 degree phase decrease
- b. A -20dB/decade decrease in gain and a +90 degree phase increase
- c. A +20dB/decade increase in gain and a -90 degree phase decrease
- d. A +20dB/decade increase in gain and a +90 degree phase increase

2. A zero in a transfer function results in:

- a. A -20dB/decade decrease in gain and a -90 degree phase decrease
- b. A -20dB/decade decrease in gain and a +90 degree phase increase
- c. A +20dB/decade increase in gain and a -90 degree phase decrease
- d. A +20dB/decade increase in gain and a +90 degree phase increase

3. What is the change in magnitude due to a pole at the pole frequency, f_p ?

- a. -20 dB
- b. -3 dB
- c. +3 dB
- d. +20 dB

Quiz: Stability 2

4. What is the phase shift due to a zero at the zero frequency, f_z ?

- a. -90°
- b. -45°
- c. $+45^\circ$
- d. $+90^\circ$

5. An amplifier circuit is unstable when $A_{ol}\beta$ is equal to

- a. 1
- b. -1
- c. 0
- d. Infinity

6. Loop gain, $A_{ol}\beta$, can be easily plotted on a Bode plot by _____.

- a. Adding the A_{ol} and $1/\beta$ curves
- b. Subtracting the A_{ol} and $1/\beta$ curves
- c. Multiplying the A_{ol} and $1/\beta$ curves
- d. Dividing the A_{ol} and $1/\beta$ curves

Quiz: Stability 2

7. Phase Margin is a measure of ____?

- a. The frequency that A_{ol} and $1/\beta$ intersect
- b. The closed loop phase of A_{cl} when $A_{cl} = 0\text{dB}$
- c. The loop-gain ($A_{ol}\beta$) phase when $A_{ol} = 0\text{dB}$
- d. The loop-gain ($A_{ol}\beta$) phase when $A_{ol}\beta = 0\text{dB}$

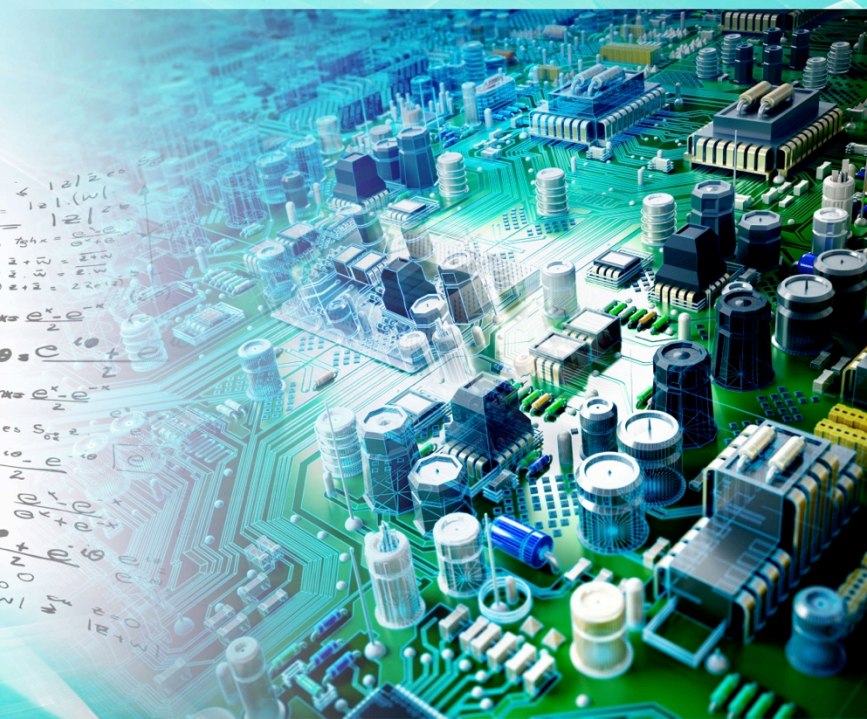
8. A rate of closure of 20dB/decade will result in a phase margin greater than ____?

- a. 30°
- b. 45°
- c. 60°
- d. 90°

Stability 2

Multiple Choice Quiz: Solutions

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Quiz: Stability 2

1. A pole in a transfer function results in:

- a. A -20dB/decade decrease in gain and a -90 degree phase decrease
- b. A -20dB/decade decrease in gain and a +90 degree phase increase
- c. A +20dB/decade increase in gain and a -90 degree phase decrease
- d. A +20dB/decade increase in gain and a +90 degree phase increase

2. A zero in a transfer function results in:

- a. A -20dB/decade decrease in gain and a -90 degree phase decrease
- b. A -20dB/decade decrease in gain and a +90 degree phase increase
- c. A +20dB/decade increase in gain and a -90 degree phase decrease
- d. A +20dB/decade increase in gain and a +90 degree phase increase

3. What is the change in magnitude due to a pole at the pole frequency, f_p ?

- a. -20 dB
- b. -3 dB
- c. +3 dB
- d. +20 dB

Quiz: Stability 2

4. What is the phase shift due to a zero at the zero frequency, f_z ?

- a. -90°
- b. -45°
- c. $+45^\circ$
- d. $+90^\circ$

5. An amplifier circuit is unstable when $A_{ol}\beta$ is equal to

- a. 1
- b. -1
- c. 0
- d. Infinity

6. Loop gain, $A_{ol}\beta$, can be easily plotted on a Bode plot by _____.

- a. Adding the A_{ol} and $1/\beta$ curves
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- d. Dividing the A_{ol} and $1/\beta$ curves

Quiz: Stability 2

7. Phase Margin is a measure of ____?

- a. The frequency that A_{ol} and $1/\beta$ intersect
- b. The closed loop phase of A_{cl} when $A_{cl} = 0\text{dB}$
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- d. The loop-gain ($A_{ol}\beta$) phase when $A_{ol}\beta = 0\text{dB}$

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- a. 30°
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- c. 60°
- d. 90°

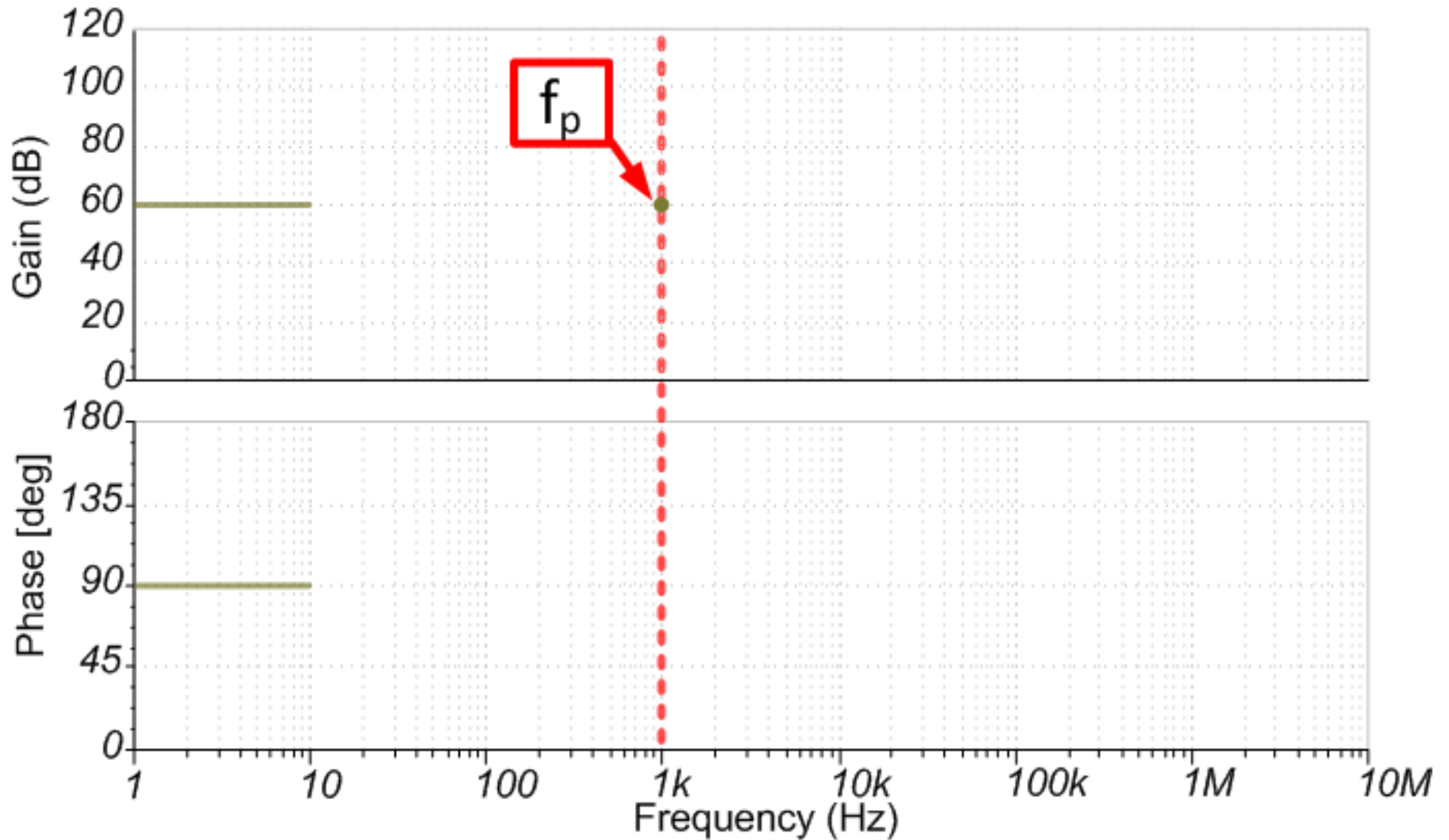
Stability 2

Exercises

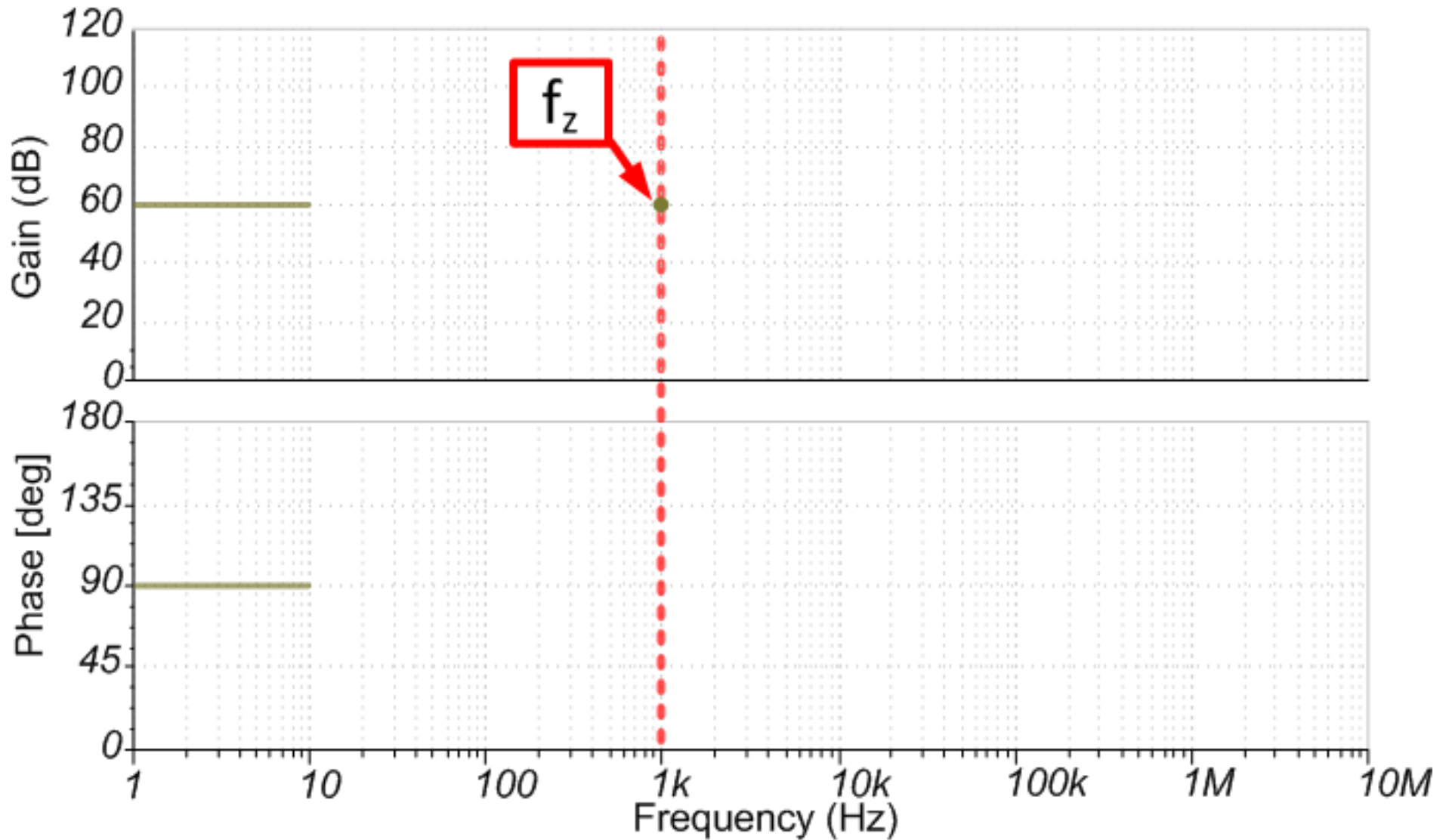
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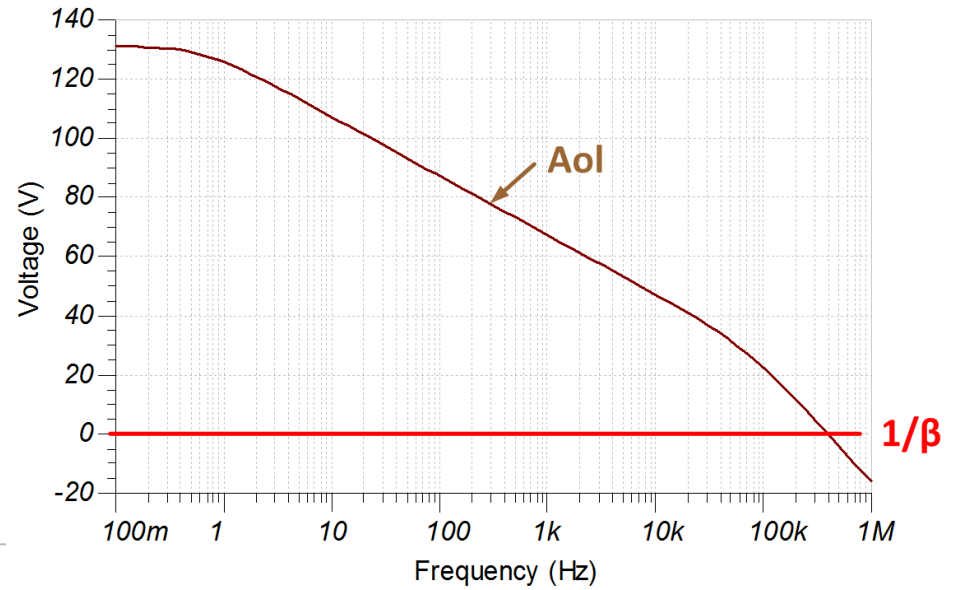
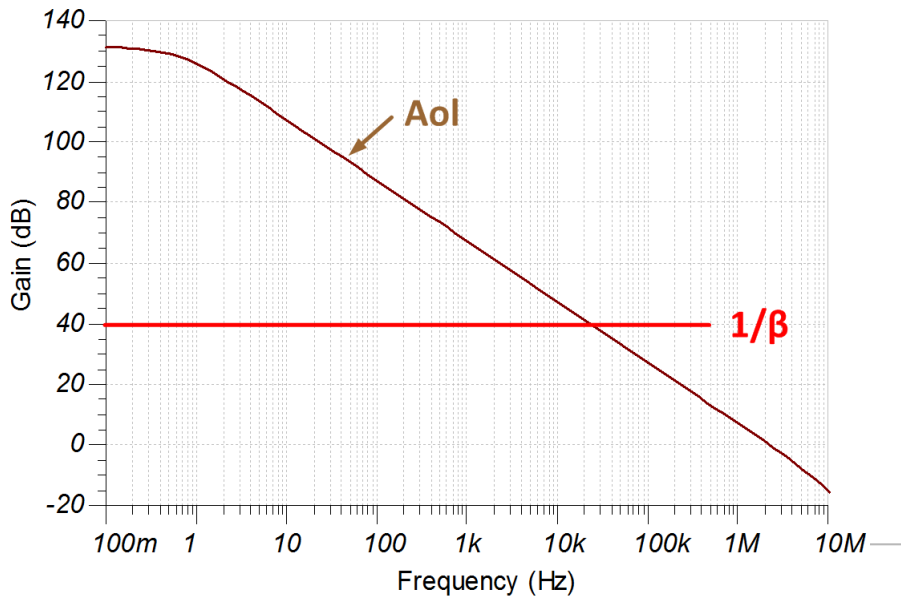
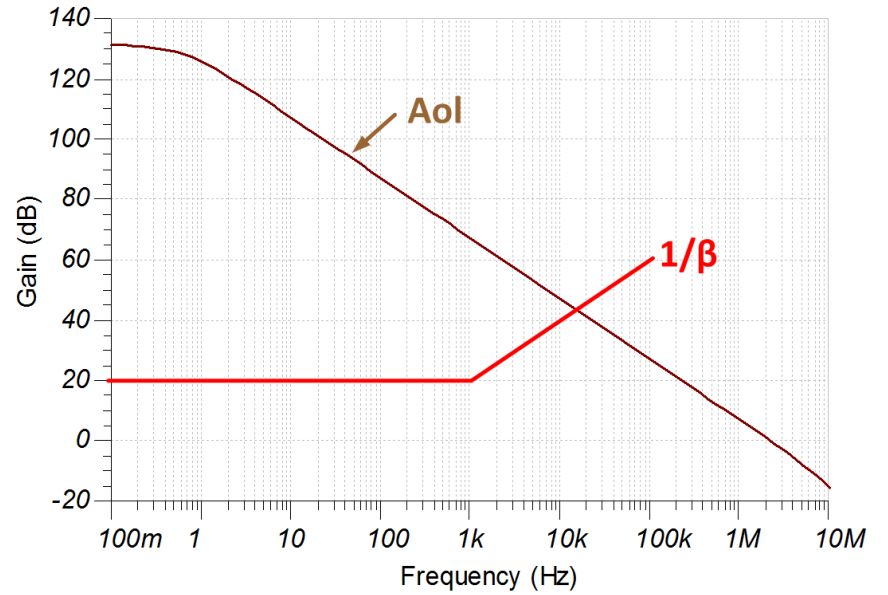
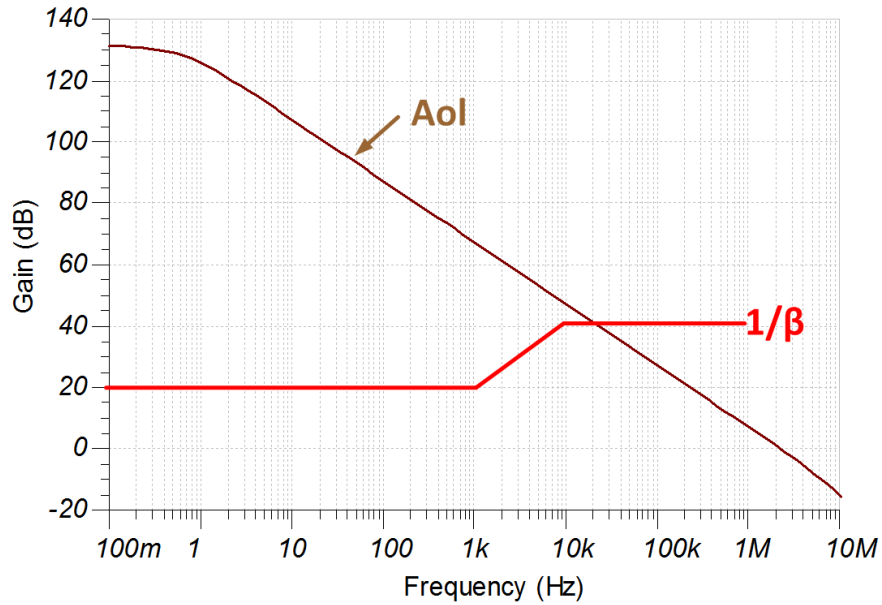
1. Finish the magnitude and phase responses assuming a pole at 1 kHz ($f_p = 1$ kHz)



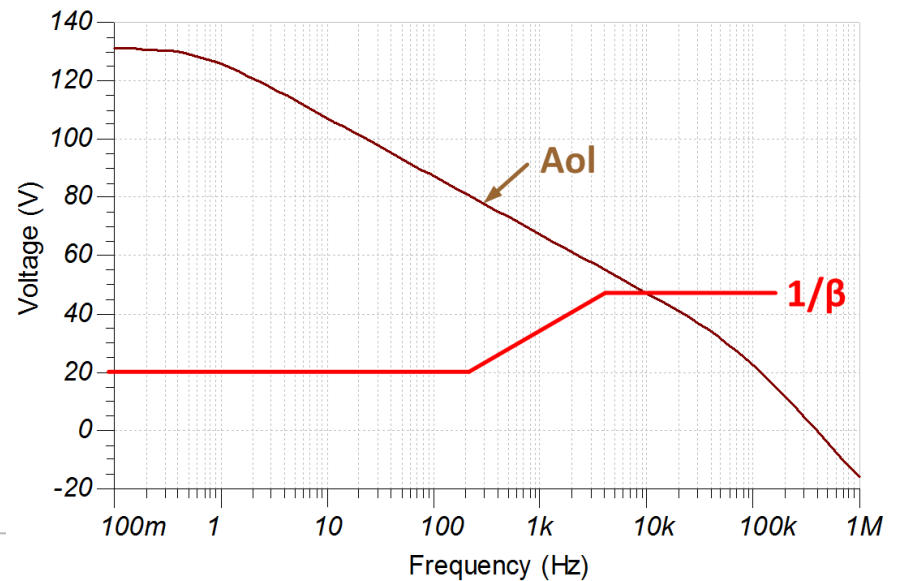
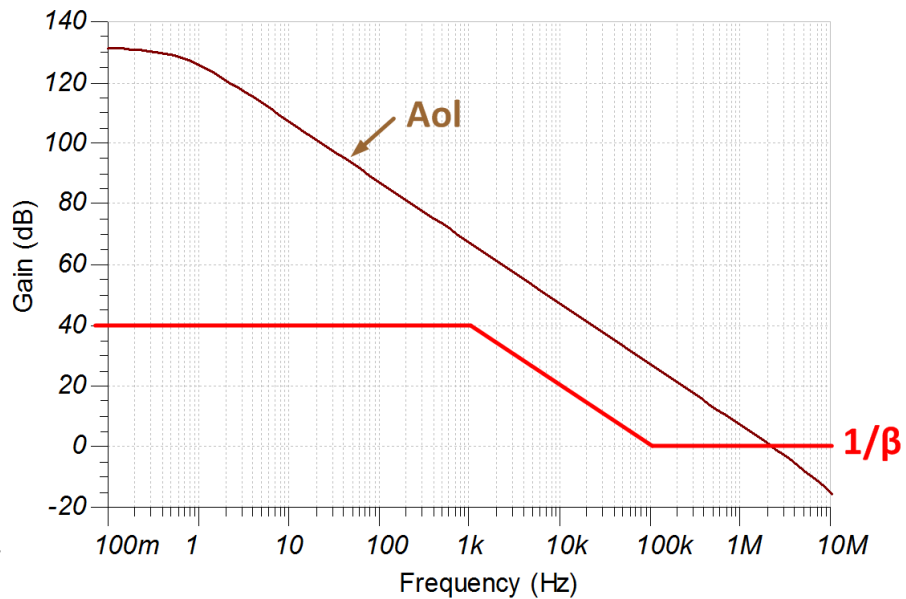
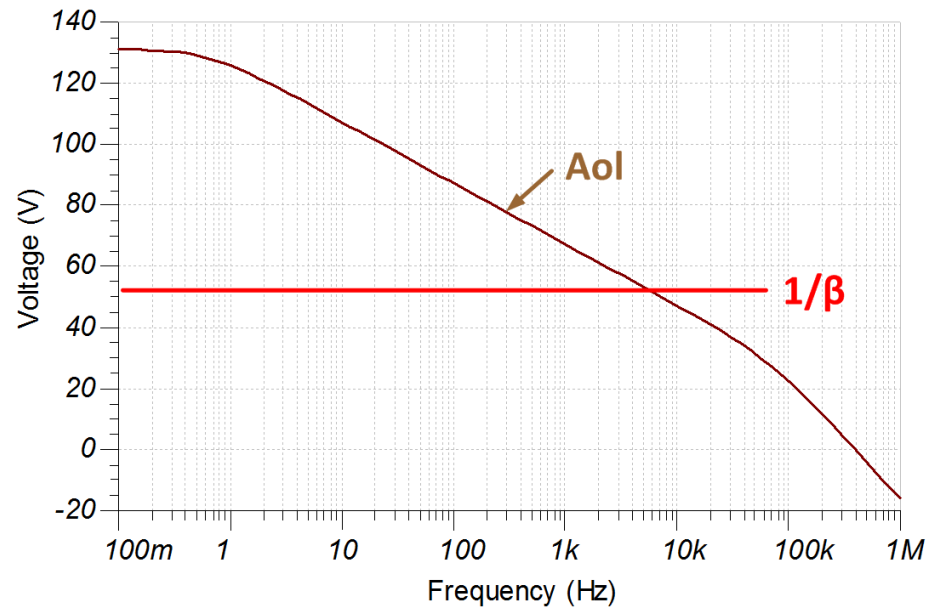
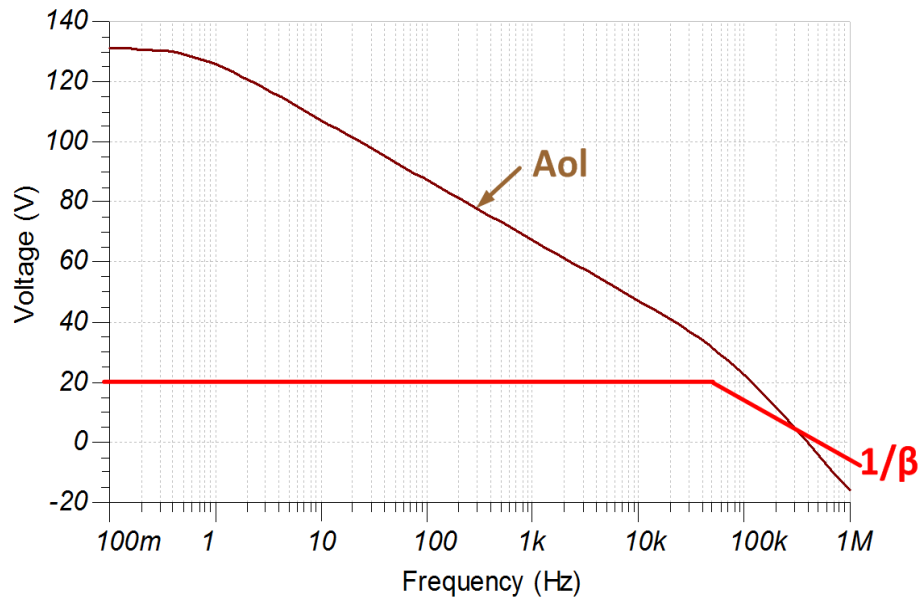
2. Finish the magnitude and phase responses assuming a zero at 1 kHz ($f_z = 1$ kHz)



3. Which of these four Aol and 1/Beta curves are stable?



4. Which of these four Aol and 1/Beta curves are stable?



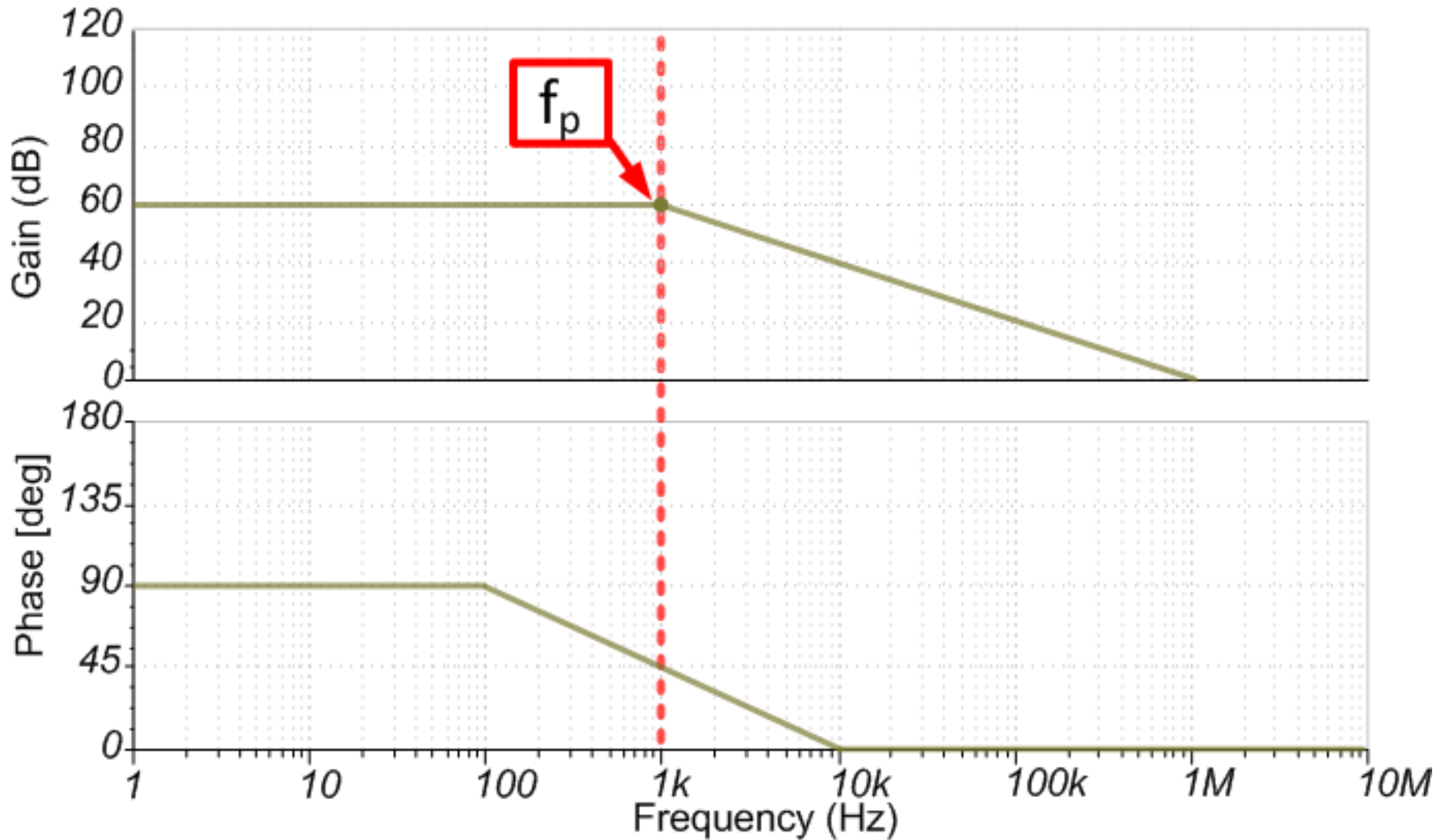
Stability 2

Solutions

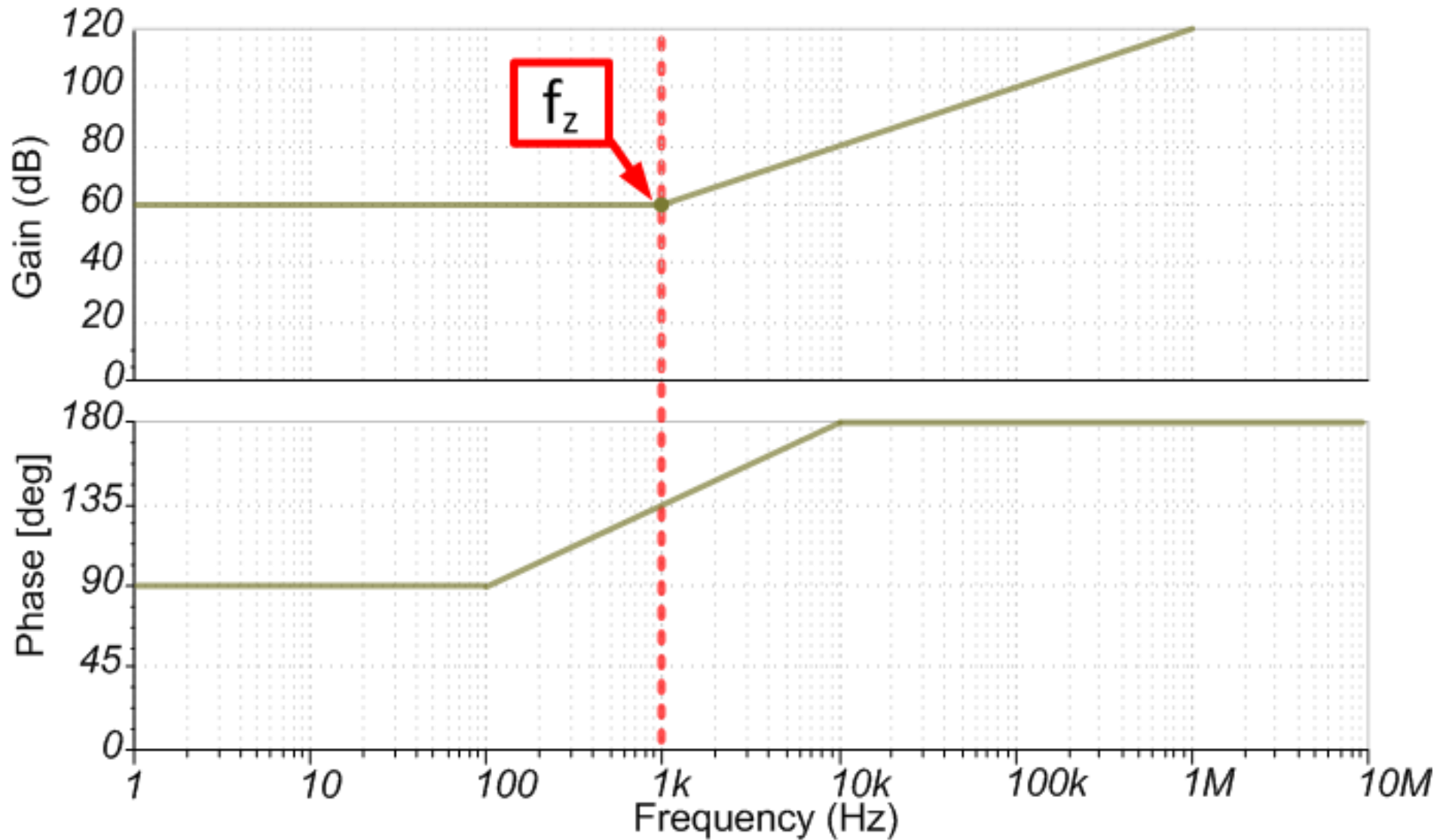
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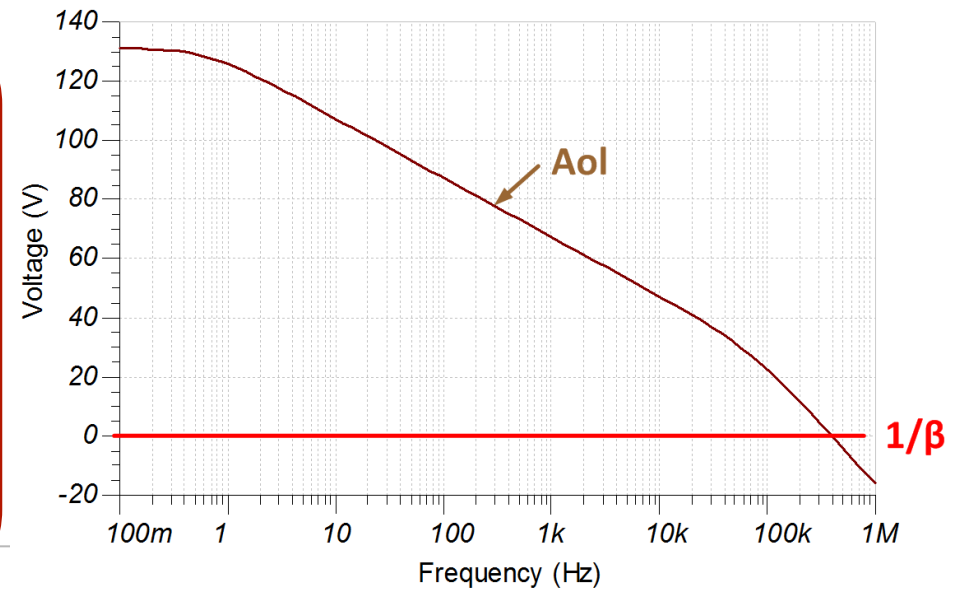
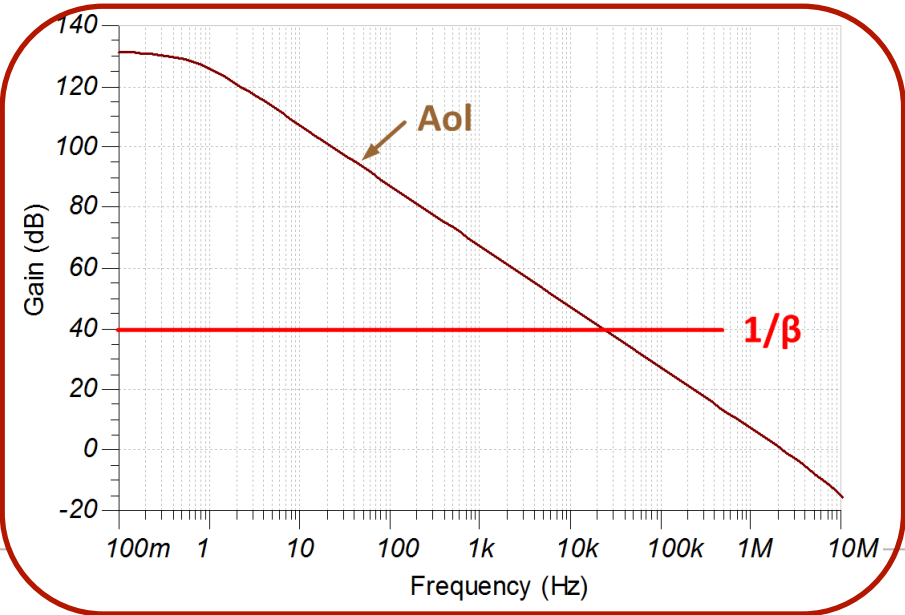
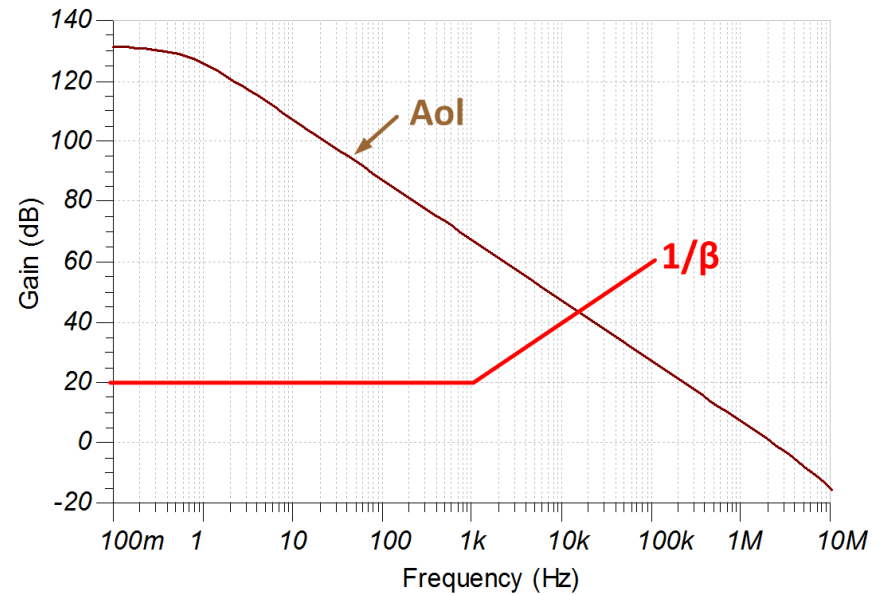
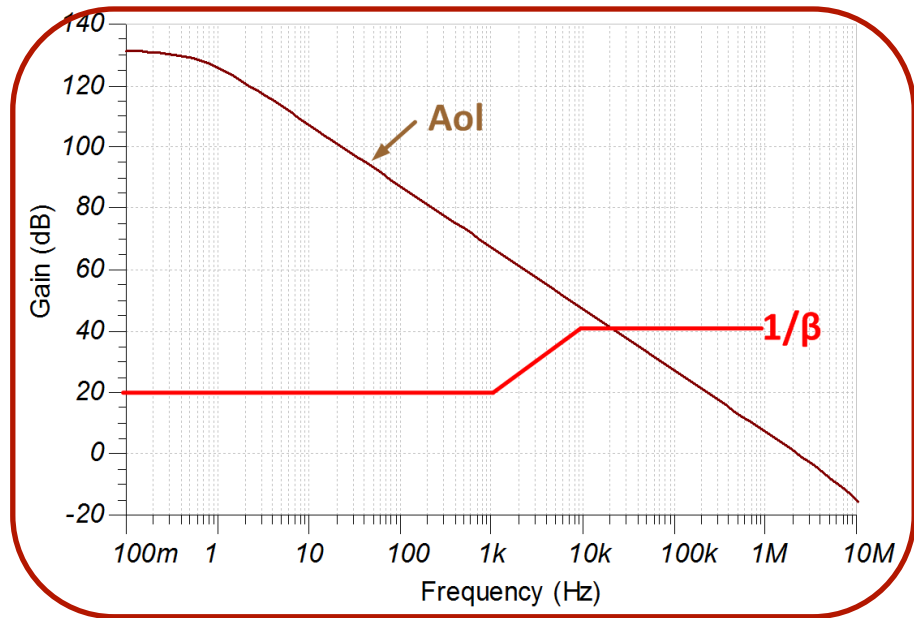
1. Finish the magnitude and phase responses assuming a pole at 1 kHz ($f_p = 1$ kHz)



2. Finish the magnitude and phase responses assuming a zero at 1 kHz ($f_z = 1$ kHz)



3. Which of these four Aol and 1/Beta curves are stable?



4. Which of these four Aol and 1/Beta curves are stable?

