Fast Fourier Transforms (FFTs) and Windowing

TIPL 4302
TI Precision Labs – ADCs

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Presented by Peggy Liska

Definition for time to frequency transformations

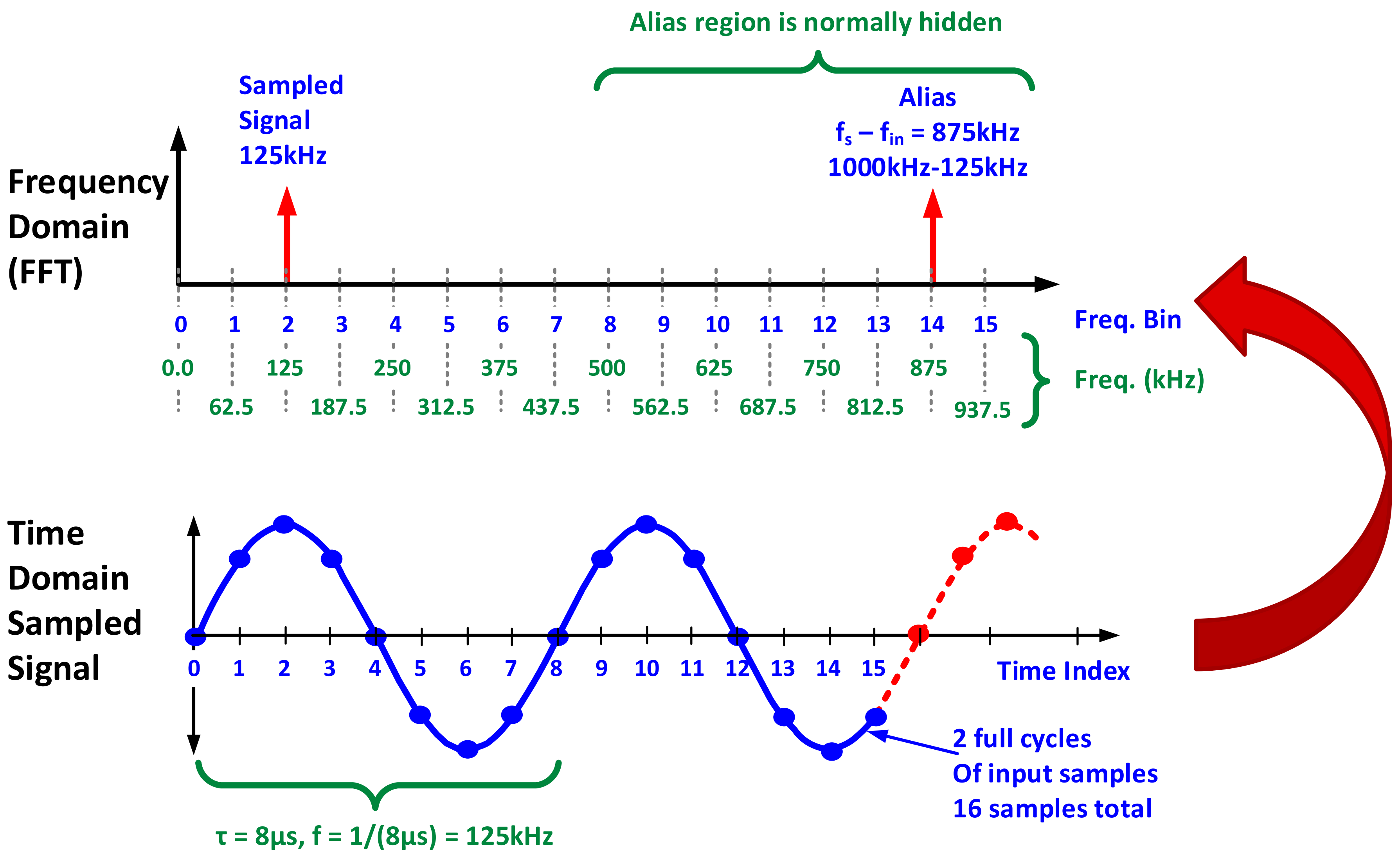
$$F(\omega) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} f[t] \cdot e^{-i\omega t} \, dt$$

Fourier Transform (continuous time)
Where
F[ω] = frequency domain continuous function
f[t] = time domain continuous function
t = time
ω = angular freqeuncy

$$F[k] = \sum_{n=0}^{N-1} f[n] \cdot e^{-i2\pi k \frac{n}{N}}$$

Fast Fourier Transform (FFT)
Used for digitized waveforms
Where
F[k] = sample in freqeuncy
f[n] = sample in time
n = time index
k = frequency index
N = number of samples
N must be a power of 2 (e.g. 256, 512, 1024…)

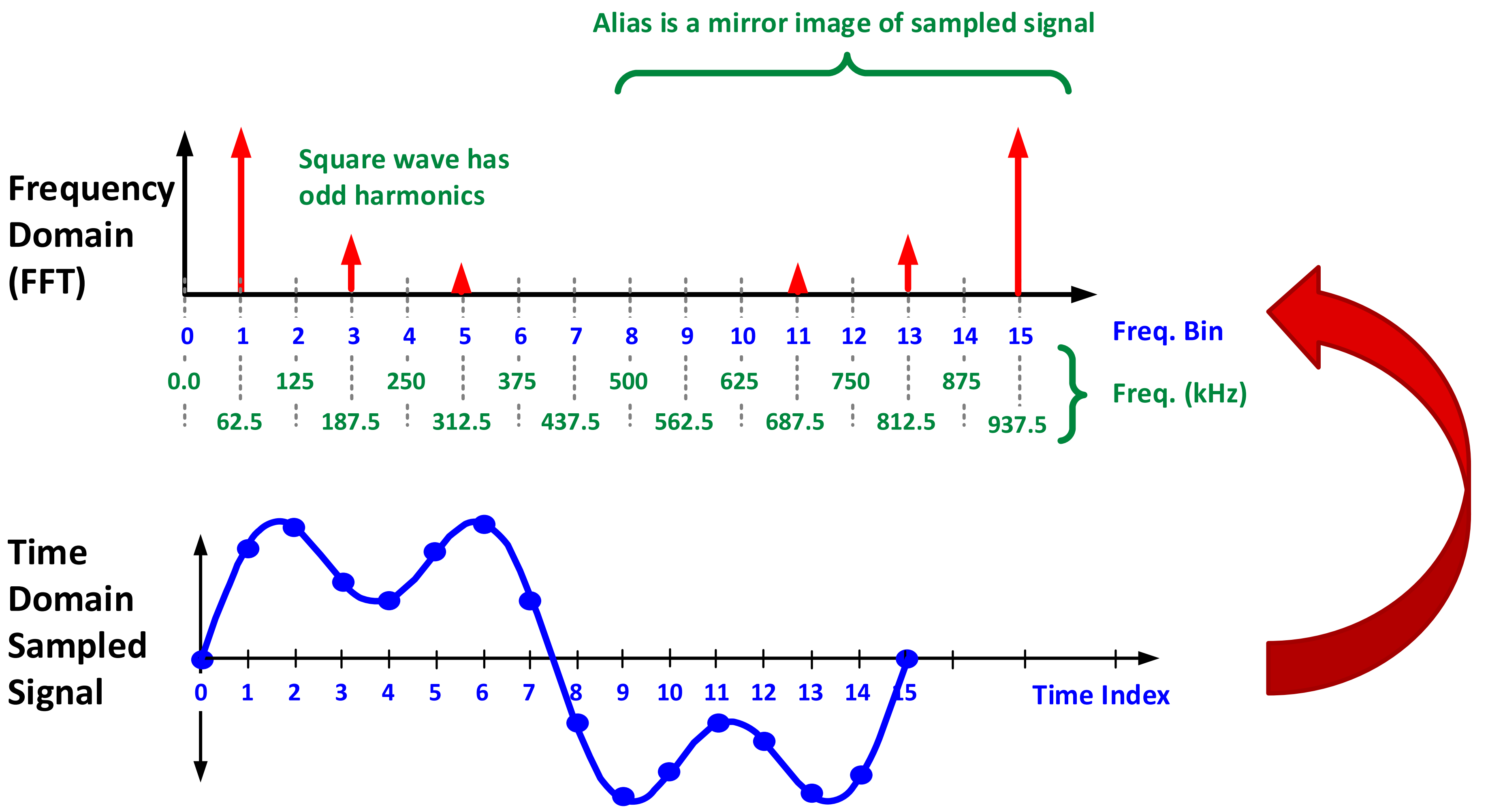
FFT Basics: Alias and Frequency Resolution



1. FFT assumes time domain continues forever
2. Number of points in time domain equals number of points in FFT
3. The alias region is normally hidden. Mirror image about fs / 2 "aliasing"
4. Frequency Resolution = Δf = fs / N

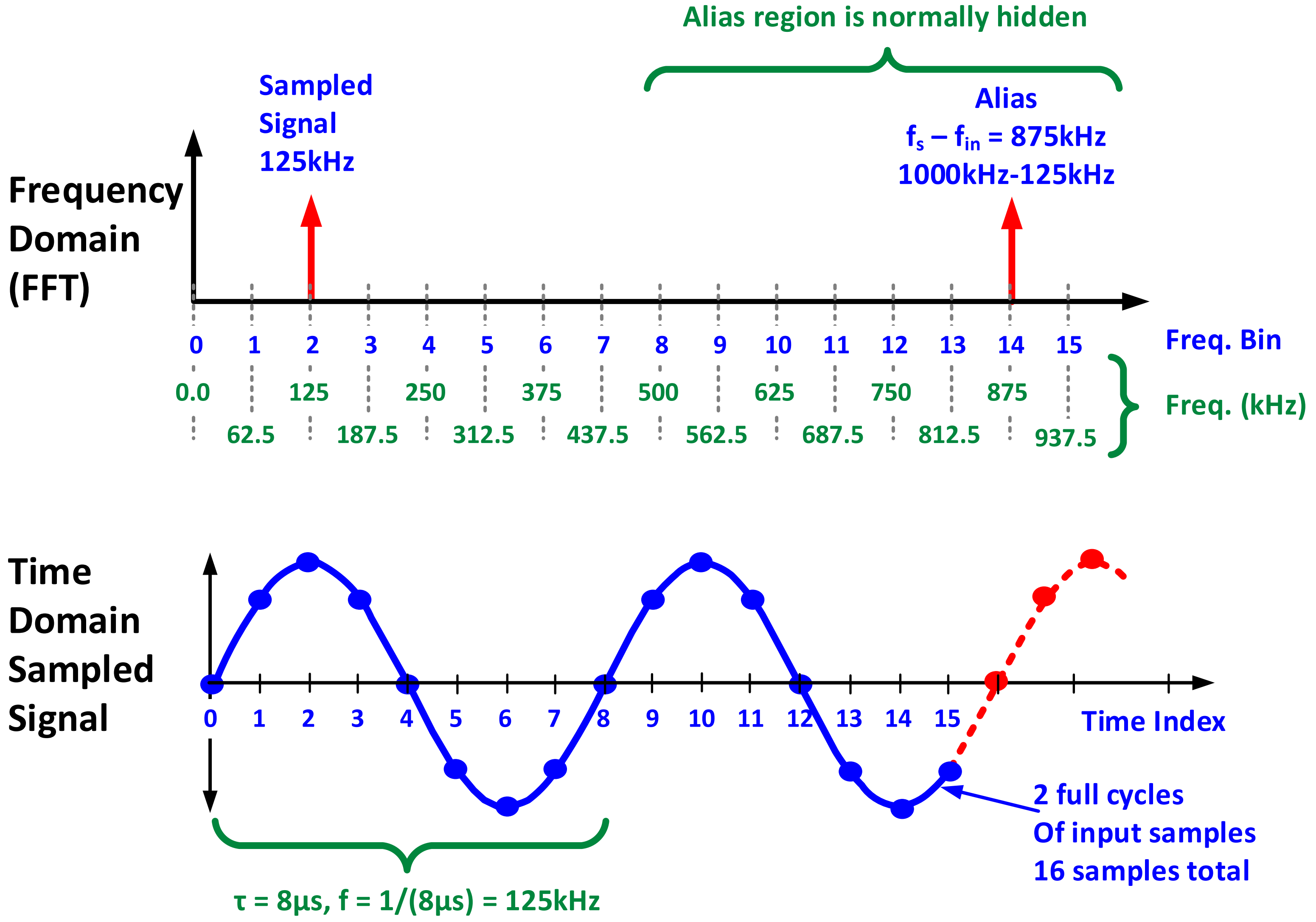
   Δf = fs / N = 1MHz / 16 = 62kHz

Alias is a Mirror Image of Sampled Signal



- Example of a “square wave”
- Square wave contains odd harmonics e.g. 3, 5, 7 …
- Notice the symmetry around 500kHz

FFT Example Calculation



Example FFT

$f_s = 1\text{MHz}$ Sampling Rate

$N_{samp} = 16$ Number of Samples

$\Delta f = \frac{f_s}{N_{samp}} = \frac{1\text{MHz}}{16} = 62.5kHz$ Frequency Resolution

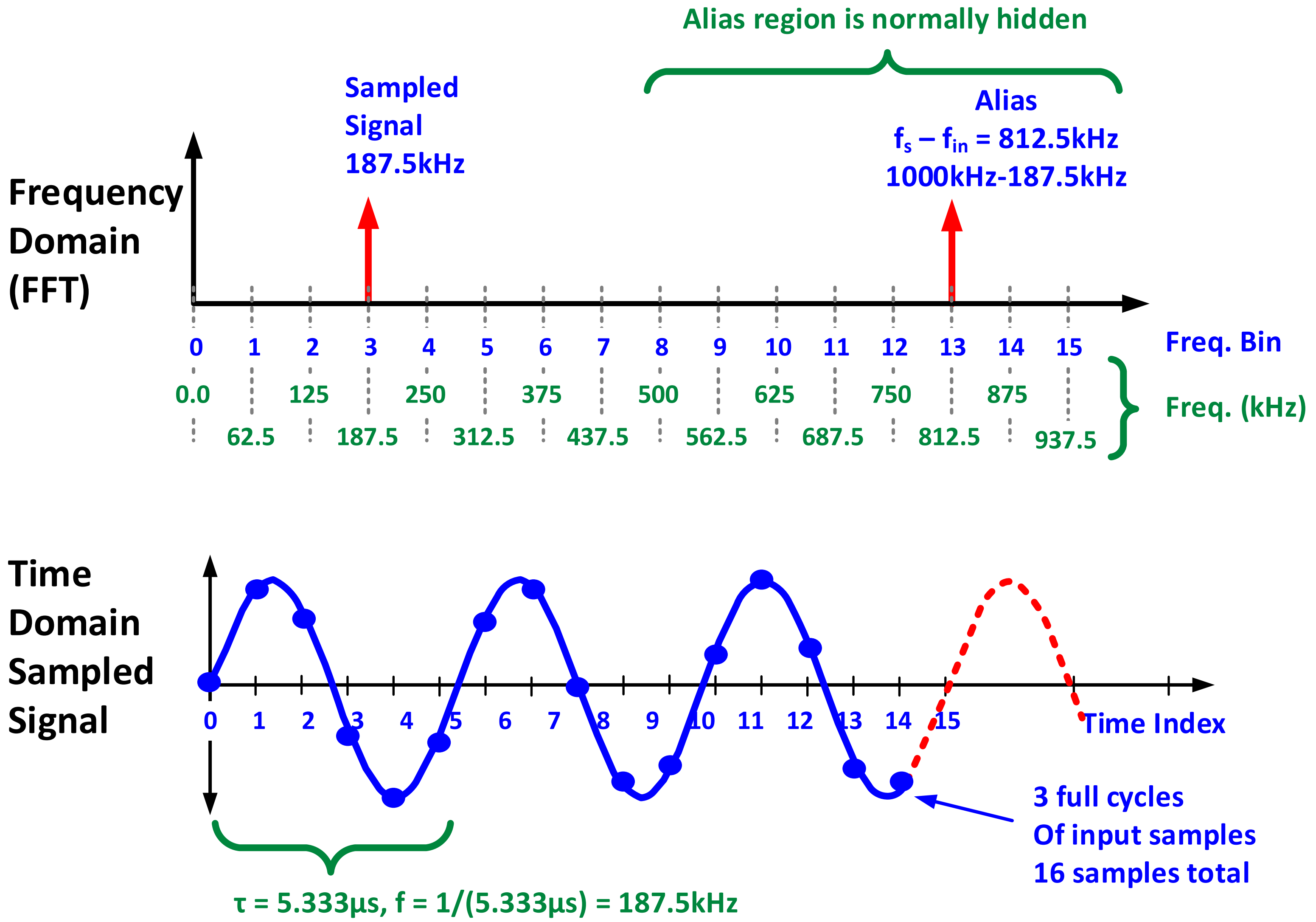
$\Delta t = \frac{1}{f_s} = \frac{1}{1\text{MHz}} = 1\mu s$ Sampling time

$f_{in} = 125kHz$ Input signal

$k_f = \frac{f_{in}}{\Delta f} = \frac{125kHz}{62.5kHz} = 2.0$ Frequency Bin

Note: f_{in} is an exact integer multiple of Δf

FFT – Different Input Frequency



Example FFT

$f_s = 1\text{Msps}$ — Sampling Rate

$N_{samp} = 16$ — Number of Samples

$$\Delta f = \frac{f_s}{N_{samp}} = \frac{1\text{Msps}}{16} = 62.5ksps$$ Frequency Resolution

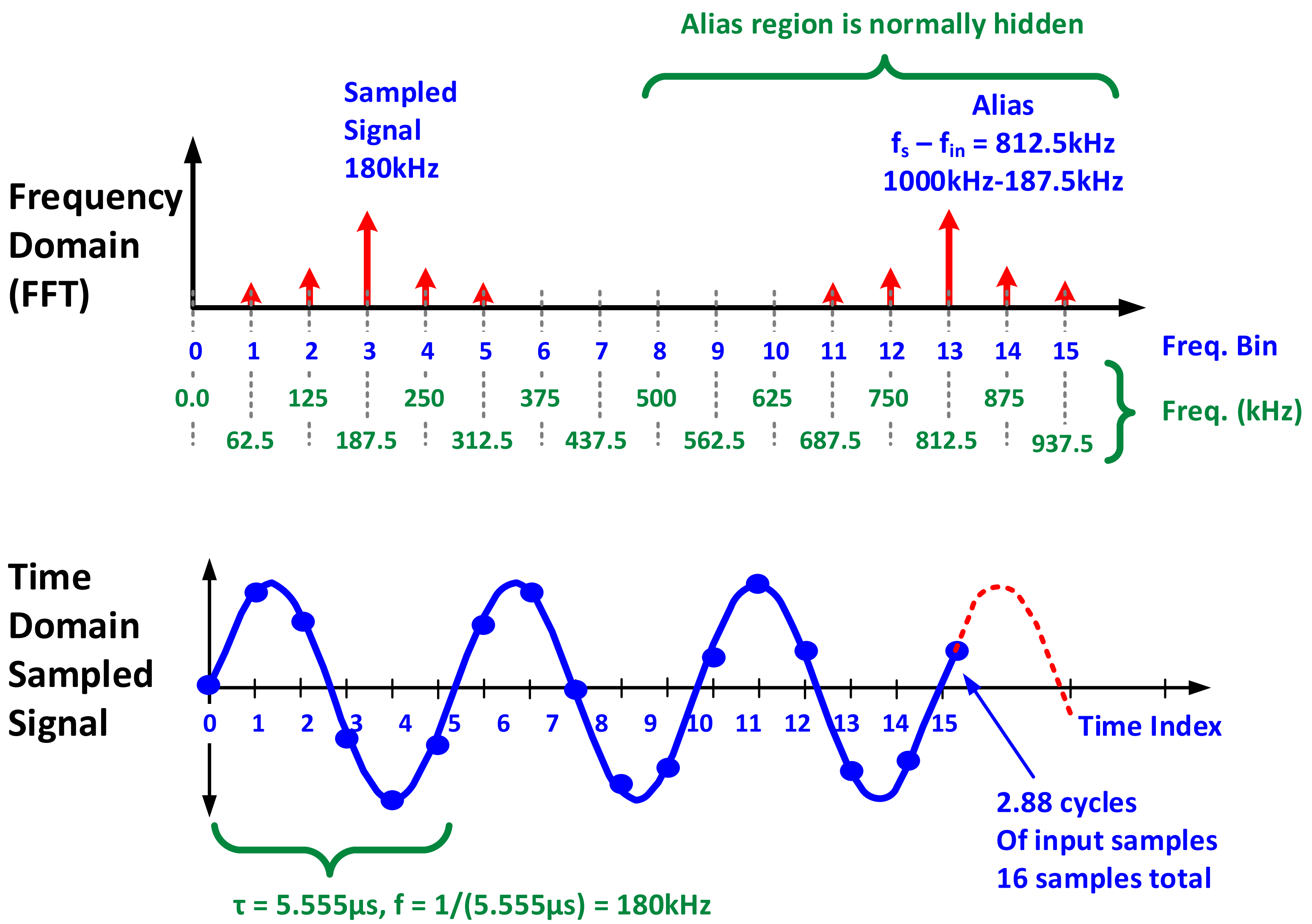
$$\Delta t = \frac{1}{f_s} = \frac{1}{1\text{Msps}} = 1\mu s$$ Sampling time

$f_{in} = 187.5kHz$ — Input signal

$$k_f = \frac{f_{in}}{\Delta f} = \frac{187.5kHz}{62.5ksps} = 3.0$$ Frequency Bin

Note: f_{in} is an exact integer multiple of Δf

FFT – Spectral Leakage



Example FFT

$f_s = 1\text{Msps}$ Sampling Rate

$N_{samp} = 16$ Number of Samples

$\Delta f = \frac{f_s}{N_{samp}} = \frac{1\text{Msps}}{16} = 62.5ksps$ Frequency Resolution

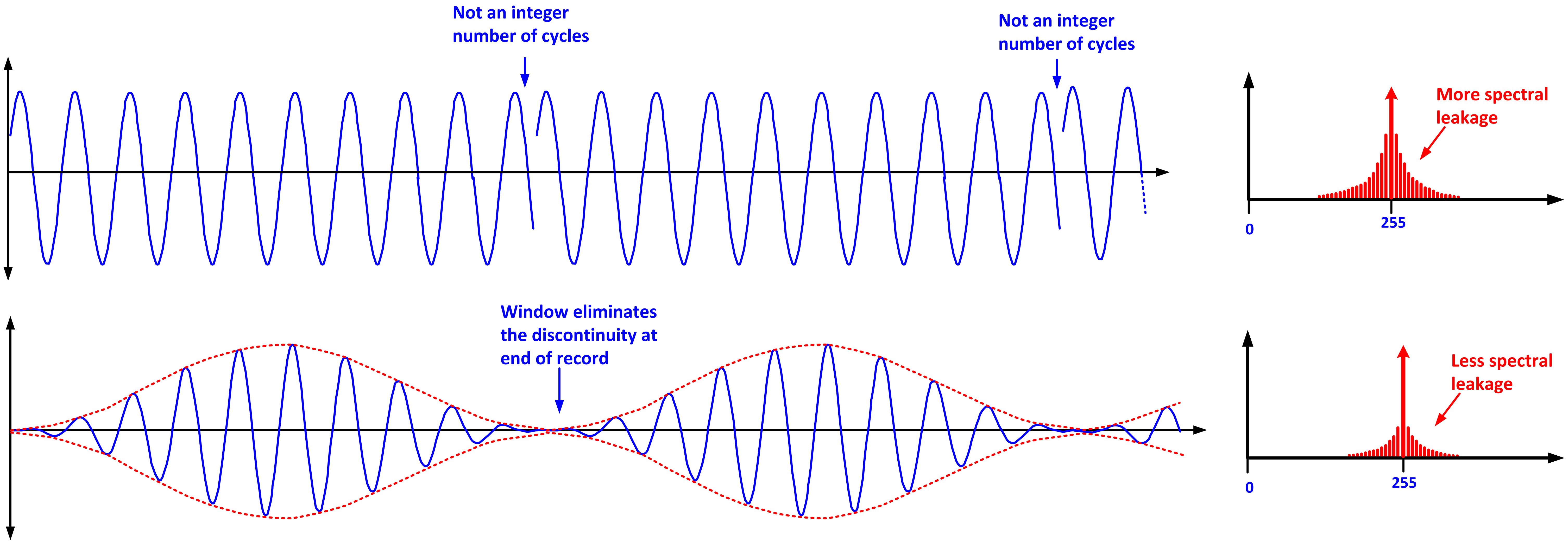
$\Delta t = \frac{1}{f_s} = \frac{1}{1\text{Msps}} = 1\mu s$ Sampling time

$f_{in} = 180kHz$ Input signal

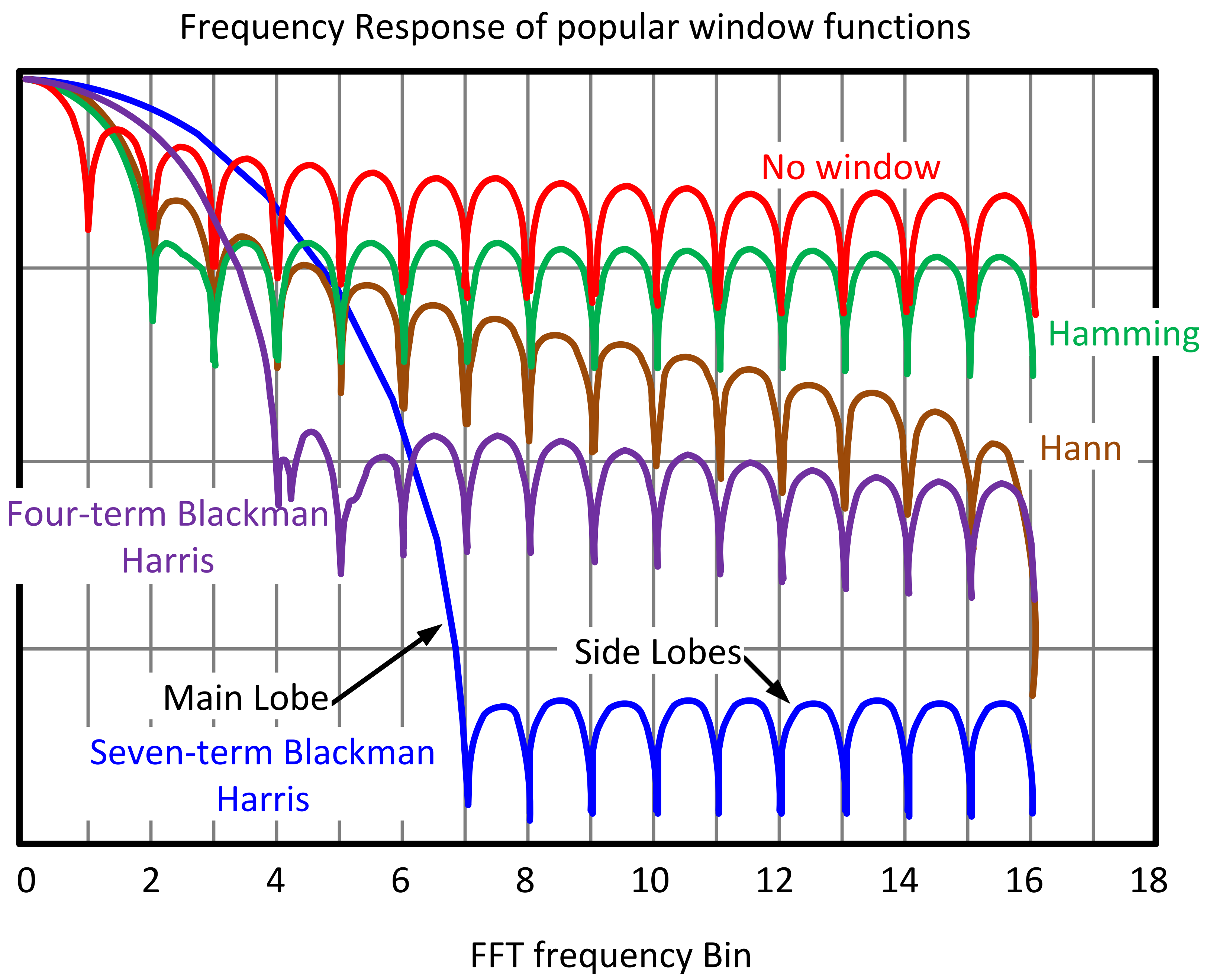
$k_f = \frac{f_{in}}{\Delta f} = \frac{180kHz}{62.5ksps} = 2.88$ Frequency Bin

Note: f_{in} is NOT an exact integer multiple of Δf

Window: Eliminates discontinuity in sampled waves



Comparing Frequency Response of Different Windows



- Ideally we would like a very narrow main lobe and very deep attenuation in side lobes.
- For ADC characterization 7 term Blackman Harris is most often used.

Different Windows for Different Applications

Signal Content	Window
ADC characterization	Blackman Harris
Sine wave or combination of waves	Hann
Sine Wave (amplitude accuracy is important)	Flat Top
Narrowband random signal (vibration data)	Hann
Broadband random (white noise)	Uniform
Closely spaced sine waves	Uniform, Hamming
Excitation signals (hammer blow)	Force
Response signals	Exponential
Unknown content	Hann
Two tones with frequencies close but amplitudes very different	Kaiser-Bessel
Two tones with frequencies close and almost equal amplitudes	Uniform
Accuracy single tone amplitude measurements	Flat Top

Window Processing Errors

Parameters used to characterize frequency response of windowing functions						
Window	Highest side lobe level (dB)	Processing loss (dB)	Scalloping loss (dB)	Worst-case processing loss (dB)	6-dB bandwidth (bins)	Half main lobe width (bins)
No window	-13	0.00	3.92	3.92	1.21	1
Hann	-32	1.76	1.33	3.09	2.00	2
Hamming	-43	1.34	1.76	3.10	1.82	2
Four-term Blackmon Harris	-92	3.00	0.83	3.83	2.72	4
Seven-term Blackmon Harris	-163	4.20	0.46	4.66	3.52	7

Thanks for your time! Please try the quiz.

Quiz: Fast Fourier Transforms (FFTs) and Windowing

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Quiz: Fast Fourier Transforms (FFTs) and Windowing

1. A time domain signal has 1024 points in it. How many points will be in the FFT?
   a) 1024.
   b) 2048.
   c) This depends on the type of FFT used.

2. The sampling rate of a particular converter is 1Msps. The FFT will contain data that extents to what frequency.
   a) 1MHz. However, the data from 500kHz to 1MHz is redundant and is normally ignored.
   b) 2MHz. However, the data from 1MHz to 2MHz is an alias.
   c) This depends on the number of points of the FFT.

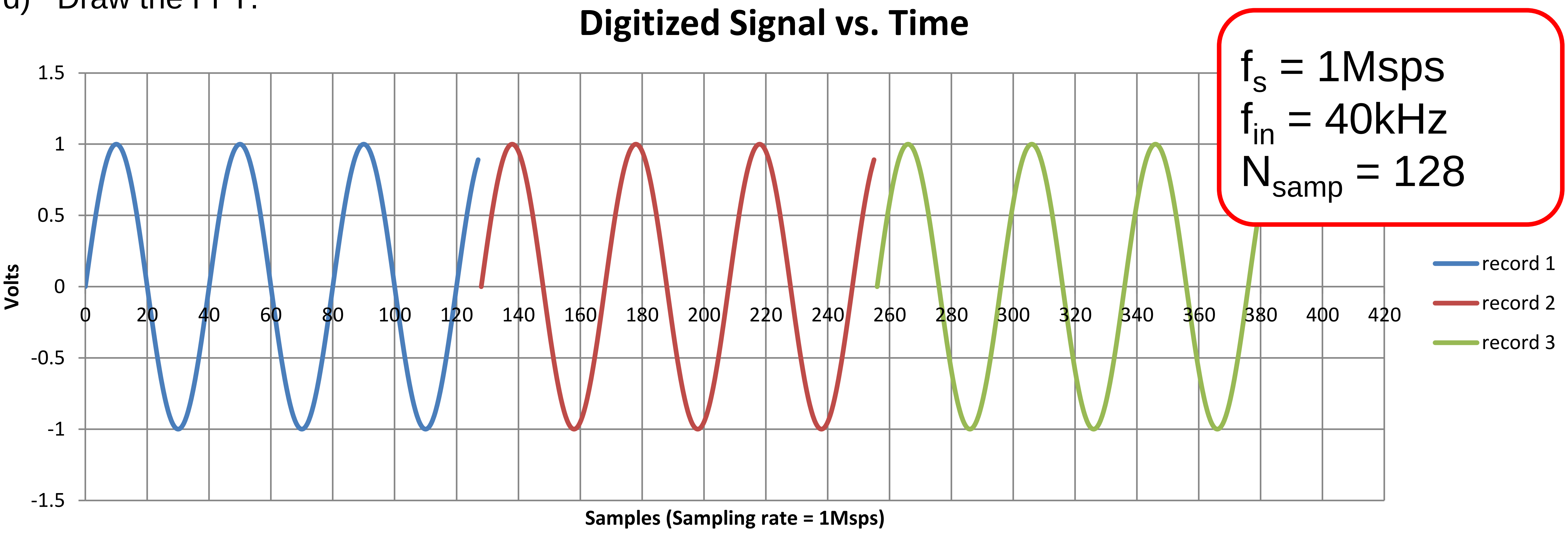
Quiz: Fast Fourier Transforms (FFTs) and Windowing

3. What is the purpose of the windowing function?
   a) The window function minimizes spectral leakage.
   b) The window function eliminates the discontinuity between records by shaping the time domain signal.
   c) The window function acts as a band stop filter to eliminate harmonic distortion.
   d) The window function is used to convert time domain signals to frequency domain signals.
   e) Both a and b are correct
   f) Both c and d are correct

4. What type of window is commonly used for ADC characterization?
   a) Hamming.
   b) Hann.
   c) The 4 term Blackman Harris.
   d) The 7 term Blackman Harris.

Quiz: Fast Fourier Transforms (FFTs) and Windowing

5. Convert the time domain signal below to its frequency domain equivalent.
   a) What is the frequency resolution?
   b) What bins does the frequency domain signal fall in?
   c) Does this FFT have spectral leakage?
   d) Draw the FFT.



f_s = 1Msps
f_{in} = 40kHz
N_{samp} = 128

Solutions

Quiz: Fast Fourier Transforms (FFTs) and Windowing

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   a) **1024**.
   b) 2048.
   c) This depends on the type of FFT used.

2. The sampling rate of a particular converter is 1Msps. The FFT will contain data that extents to what frequency.
   a) **1MHz. However, the data from 500kHz to 1MHz is redundant and is normally ignored.**
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    f) Both c and d are correct

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    c) The 4 term Blackman Harris.
    d) **The 7 term Blackman Harris.**

Quiz: Fast Fourier Transforms (FFTs) and Windowing

5. Convert the time domain signal below to its frequency domain equivalent.
   a) What is the frequency resolution? **$\Delta f = f_s/N_{samp}$ = 1MHz/128 = 7.8125kHz**
   b) What bins does the frequency domain signal fall in? **$k_f = f_{in}/\ \Delta f$ = 40kHz/7.8125kHz = 5.12**
   c) Does this FFT have spectral leakage? **Yes. You can clearly see the discontinuity at the end of each record.**
   d) Draw the FFT.