Application Report
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Modeling Bi-Directional Buck/Boost Converter for Digital Control Using C2000 Microcontrollers

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ABSTRACT

This application report derives a unified model of a bi-directional buck boost converter, in either mode of operation, using state space averaging technique.

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1 Introduction

Bi-directional power flow converters are gaining interest because of popularity of renewable and electric vehicles. State space averaging method is used to derive a unified model of a buck boost converter and relevant transfer functions are derived for control of the voltage and current. Digital control using microcontrollers such as Texas Instruments C2000™ platform is widely used in such applications because of flexibility of software, which enables implementing current sharing and robust control under varying conditions easy.

A typical application for such converters is hybrid vehicles, where multiple energy sources are combined to provide a stable bus for the motor drive, and in case of regenerative breaking, recuperate that energy by storing it for future use. Depending on storage characteristics of individual storage elements control of the current or the voltage is desired. For example, the ultra capacitor in the converter, because of its fast charge and discharge, is used to maintain the bus voltage constant during transients of the drive and voltage mode control is used (see Figure 1). Whereas, for the battery and the fuel cell elements, the current control mode is used. A generic model of the power stage is developed using state space averaging to help analyze control of output voltage, and the input current and MATLAB script is provided to simulate the power stage.

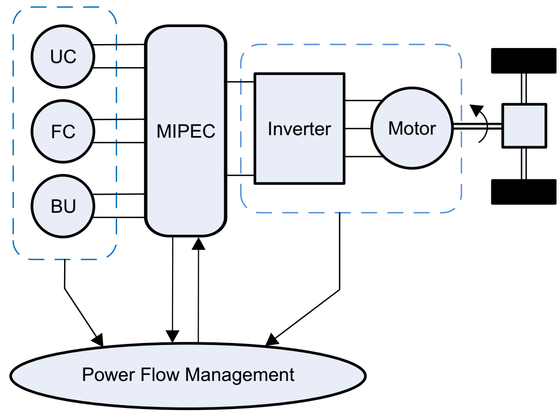

Figure 1. Hybrid Vehicle Using Multiple Input Energy Sources

2 Power Stage Definition

To facilitate the model derivation, the power stage is defined as shown in Figure 2. The different components of the power stage are:

- Input voltage source voltage, V_p
- Input internal source resistance, R_p
- Input source current, i_p
- Input and output capacitors, C_i, C_o
- Input and output capacitors ESR, R_{Ci}, R_{Co}
- Voltage and current in the input and output capacitor, V_{Ci}, V_{Co}, i_{Ci}, i_{Co}
- Power inductor and DCR of the inductor, L, R_L
- Voltage across and current in the inductor, V_L, i_L
- Output voltage and output load current, V_{out}, I_o
- Power switches Q1 and Q2

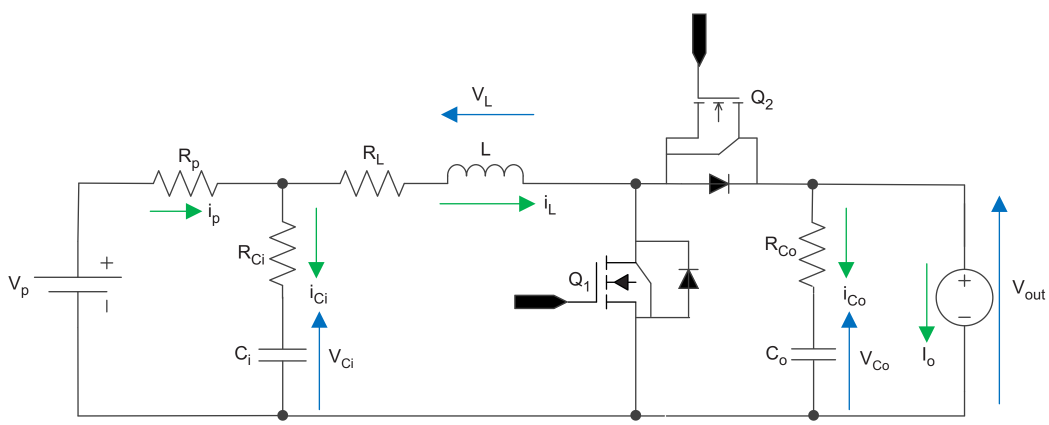

Figure 2. Synchronous Boost Stage With Energy Storage Elements at Input and Output

3 State Space Modeling

State space averaging is commonly applied to develop models for switched mode power supplies (SMPS). It is common to choose the energy storage elements as the state of the system, which is current for inductors and voltage for capacitors. It is noted irrespective of charging or discharging, that is buck or boost, as the switches Q1 and Q2 are complimentary to each other and there are only two sub-intervals: State 1 (Q1 on Q2 off) and State 2 (Q1 off, Q2 on). Therefore, a common model can be developed for both the modes of power flow.

3.1 State Selection

For the DC-DC converter, shown in Figure 2, state (X) , input (Ui) and output (Y) vectors are chosen as follows:

$$X = \begin{bmatrix} Output\_Cap\_Voltage \\ Input\_Cap\_Voltage \\ Inductor\_Current \end{bmatrix} = \begin{bmatrix} v_{Co} \\ v_{Ci} \\ i_L \end{bmatrix}$$

$$U_i = \begin{bmatrix} Output\_Load \\ Input\_Voltage \end{bmatrix} = \begin{bmatrix} I_o \\ V_p \end{bmatrix}$$

$$Y = \begin{bmatrix} Input\_Current \\ Inductor\_Current \\ Output\_Voltage \end{bmatrix} = \begin{bmatrix} i_p \\ i_L \\ V_{out} \end{bmatrix}$$

The energy storage elements are chosen as states (voltage of the capacitor and current in the inductor). The output load and input voltage are considered as the inputs. The output voltage, input current and the inductor current are chosen as the controlled variables.

3.1.1 State 1: Q1 ON, Q2 OFF

Figure 3 shows the power stage in State 1. Using KCL and KVL equations at the highlighted nodes, Equation 1 through Equation 5 can be written.

$$C_O \frac{dV_{Co}}{dt} = -I_O \tag{1}$$

$$i_p = i_L + i_{Ci} \tag{2}$$

$$V_p - i_p R_p - i_{Ci} R_{Ci} - V_{Ci} = 0 \tag{3}$$

$$V_{Ci} + i_{Ci} R_{Ci} - i_L R_L - L \frac{di_L}{dt} = 0 \tag{4}$$

$$i_{Ci} = C_i \frac{dV_{Ci}}{dt} \tag{5}$$

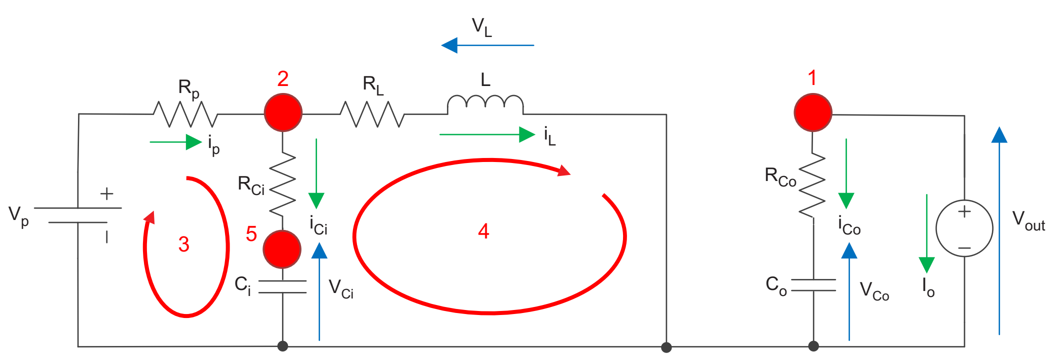

Figure 3. State 1, Q1 ON

Re-arranging the equations, the state of the system can be expressed as shown in Equation 6:

$$\dot{X} = A_1X + B_1U_i = \begin{bmatrix} 0 & 0 & 0 \\ 0 & -\frac{1}{C_i(R_p + R_{Ci})} & -\frac{R_p}{C_i(R_p + R_{Ci})} \\ 0 & \frac{R_p}{L(R_p + R_{Ci})} & -\frac{(R_L R_p + R_L R_{Ci} + R_{Ci} R_p)}{L(R_p + R_{Ci})} \end{bmatrix} \begin{bmatrix} V_{Co} \\ V_{Ci} \\ i_L \end{bmatrix} + \begin{bmatrix} -\frac{1}{C_o} & 0 \\ 0 & \frac{1}{C_i(R_p + R_{Ci})} \\ 0 & \frac{R_{ci}}{L(R_p + R_{Ci})} \end{bmatrix} \begin{bmatrix} I_o \\ V_p \end{bmatrix} \tag{6}$$

Similarly, the output state equation can be expressed as shown in Equation 7:

$$Y = C_1X + E_1U_i = \begin{bmatrix} i_p \\ i_L \\ V_{out} \end{bmatrix} = \begin{bmatrix} 0 & -\frac{1}{(R_{Ci} + R_p)} & \frac{R_{ci}}{(R_{Ci} + R_p)} \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} V_{Co} \\ V_{Ci} \\ i_L \end{bmatrix} + \begin{bmatrix} 0 & \frac{1}{(R_{Ci} + R_p)} \\ 0 & 0 \\ -R_{Co} & 0 \end{bmatrix} \begin{bmatrix} I_o \\ V_p \end{bmatrix} \tag{7}$$

3.1.2 State 2: Q1 OFF, Q2 ON

Figure 4 shows the power stage in State 1. Using KCL and KVL equations at the highlighted nodes, Equation 8 through Equation 12 can be written.

$$I_o = i_L - i_{Co} = i_L - C_o \frac{dV_{Co}}{dt} \tag{8}$$

$$i_p = i_L + i_{Ci} \tag{9}$$

$$V_p - i_p R_p - i_{Ci} R_{Ci} - V_{Ci} = 0 \tag{10}$$

$$i_{Ci} = C_i \frac{dV_{Ci}}{dt} \tag{11}$$

$$V_p - i_p R_p - i_L R_L - L\frac{di_L}{dt} - R_{Co} i_{Co} - V_{Co} = 0 \tag{12}$$

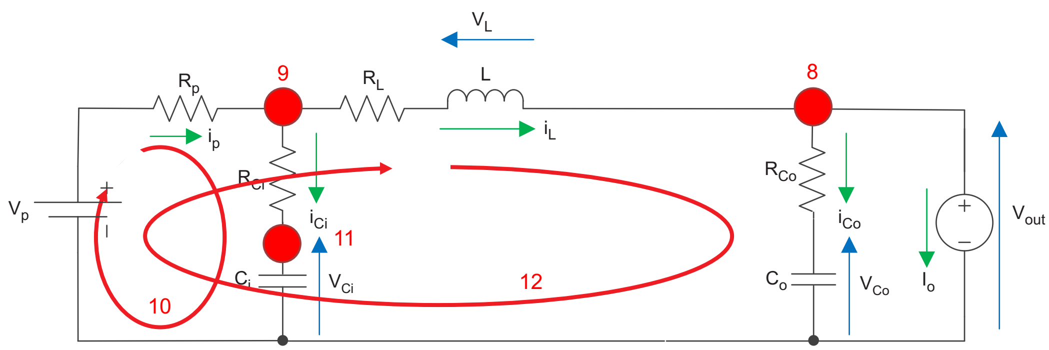

Figure 4. State 2, Q1 OFF

Re-arranging the equations, the state of the system can be expressed as shown in Equation 13:

$$\dot{X} = A_2X + B_2U_i = \begin{bmatrix} 0 & 0 & \frac{1}{C_o} \\ 0 & -\frac{1}{C_i(R_p + R_{Ci})} & -\frac{R_p}{C_i(R_p + R_{Ci})} \\ -\frac{1}{L} & \frac{R_p}{L(R_p + R_{Ci})} & -\frac{(R_L R_p + R_L R_{Ci} + R_{Ci} R_P + R_p R_{Co} + R_{Co} R_{Ci})}{L(R_p + R_{Ci})} \end{bmatrix} \begin{bmatrix} V_{Co} \\ V_{Ci} \\ i_L \end{bmatrix} + \begin{bmatrix} -\frac{1}{C_o} & 0 \\ 0 & \frac{1}{C_i(R_p + R_{Ci})} \\ \frac{R_{Co}}{L} & \frac{R_{Ci}}{L(R_p + R_{Ci})} \end{bmatrix} \begin{bmatrix} I_o \\ V_p \end{bmatrix} \tag{13}$$

Similarly, the output state equation can be expressed as shown in Equation 14:

$$Y = C_2X + E_2U_i = \begin{bmatrix} i_p \\ i_L \\ V_{out} \end{bmatrix} = \begin{bmatrix} 0 & -\frac{1}{(R_{Ci}+R_p)} & \frac{R_{Ci}}{(R_{Ci}+R_p)} \\ 0 & 0 & 1 \\ 1 & 0 & R_{Co} \end{bmatrix} \begin{bmatrix} V_{Co} \\ V_{Ci} \\ i_L \end{bmatrix} + \begin{bmatrix} 0 & \frac{1}{(R_{Ci}+R_p)} \\ 0 & 0 \\ -R_{Co} & 0 \end{bmatrix} \begin{bmatrix} I_o \\ V_p \end{bmatrix} \tag{14}$$

3.2 Averaged Large Signal Model

Next, an averaged large signal model is derived for this. Assuming that State 1 is for duty cycle D, the averaged state space equation can be written as shown in Equation 15:

$$\dot{X} = (A_1D + A_2(1-D))X + (B_1D + B_2(1-D))U_i$$

$$= \begin{bmatrix} 0 & 0 & (1-D)/C_o \\ 0 & -\frac{1}{C_i(R_p+R_{Ci})} & -\frac{R_p}{C_i(R_p+R_{Ci})} \\ -(1-D)\frac{1}{L} & \frac{R_p}{L(R_p+R_{Ci})} & -\frac{(R_LR_p+R_LR_{Ci}+R_{Ci}R_P+(1-D)(R_pR_{Co}+R_{Co}R_{Ci}))}{L(R_p+R_{Ci})} \end{bmatrix} \begin{bmatrix} V_{Co} \\ V_{Ci} \\ i_L \end{bmatrix} + \begin{bmatrix} -\frac{1}{C_o} & 0 \\ 0 & \frac{1}{C_i(R_p+R_{Ci})} \\ (1-D)\frac{R_{Co}}{L} & \frac{R_{Ci}}{L(R_p+R_{Ci})} \end{bmatrix} \begin{bmatrix} I_o \\ V_p \end{bmatrix} \tag{15}$$

Similarly, the output state equation can be expressed as shown in Equation 16:

$$Y = (C_1D + C_2(1-D))X + (E_1D + E_2(1-D))U_i$$

$$\Rightarrow \begin{bmatrix} i_p \\ i_L \\ V_{out} \end{bmatrix} = \begin{bmatrix} 0 & -\frac{1}{(R_{Ci}+R_p)} & \frac{R_{Ci}}{(R_{Ci}+R_p)} \\ 0 & 0 & 1 \\ 1 & 0 & (1-D)R_{Co} \end{bmatrix} \begin{bmatrix} V_{Co} \\ V_{Ci} \\ i_L \end{bmatrix} + \begin{bmatrix} 0 & \frac{1}{(R_{Ci}+R_p)} \\ 0 & 0 \\ -R_{Co} & 0 \end{bmatrix} \begin{bmatrix} I_o \\ V_p \end{bmatrix} \tag{16}$$

Assuming small perturbation on the state, duty, and input vectors, the small signal and steady state models can be separated where the steady state value is denoted by bar and the small perturbation is denoted by the hat symbol.

$$X = \bar{X} + \hat{X} \tag{17}$$

$$U_i = \bar{U}_i + \hat{U}_i \tag{18}$$

$$Y = \bar{Y} + \hat{Y} \tag{19}$$

$$D = \bar{D} + \hat{d} \tag{20}$$

$$V_p = \bar{V}_p + \hat{V}_p \tag{21}$$

$$I_o = \bar{I}_o + \hat{i}_o \tag{22}$$

$$V_{Ci} = \bar{V}_{Ci} + \hat{V}_{Ci} \tag{23}$$

$$V_{Co} = \bar{V}_{Co} + \hat{V}_{Co} \tag{24}$$

$$i_L = \bar{i}_L + \hat{i}_L \tag{25}$$

$$i_p = \bar{i}_p + \hat{i}_p \tag{26}$$

$$V_{out} = \bar{V}_{out} + \hat{V}_{out} \tag{27}$$

3.3 Steady State Model

Separating the steady state terms from the large signal model Equation 16 by using Equation 17 through Equation 27, you can write Equation 28.

$$\dot{X} = (A_1\bar{D} + A_2(1-\bar{D}))\bar{X} + (B_1\bar{D} + B_2(1-\bar{D}))\bar{U}_i = 0$$

$$= \begin{bmatrix} 0 & 0 & \frac{(1-\bar{D})}{C_O} \\ 0 & -\frac{1}{C_i(R_p+R_{Ci})} & -\frac{R_p}{C_i(R_p+R_{Ci})} \\ -(1-\bar{D})\frac{1}{L} & \frac{R_p}{L(R_p+R_{Ci})} & -\frac{(R_L R_p + R_L R_{Ci} + R_{Ci} R_p + (1-\bar{D})(R_p R_{Co} + R_{Co} R_{Ci}))}{L(R_p+R_{Ci})} \end{bmatrix} \begin{bmatrix} \bar{V}_{Co} \\ \bar{V}_{Ci} \\ \bar{i}_L \end{bmatrix} + \begin{bmatrix} -\frac{1}{C_O} & 0 \\ 0 & \frac{1}{C_i(R_p+R_{Ci})} \\ (1-D)\frac{R_{Co}}{L} & \frac{R_{Ci}}{L(R_p+R_{Ci})} \end{bmatrix} \begin{bmatrix} \bar{I}_O \\ \bar{V}_p \end{bmatrix} \tag{28}$$

From Equation 28, the steady state current and voltage equations can be derived (see Equation 29 through Equation 33).

$$\bar{i}_p = \bar{i}_L = \frac{\bar{I}_O}{(1-\bar{D})} \tag{29}$$

$$\bar{i}_p = \bar{i}_L = \frac{\bar{I}_O}{(1-\bar{D})} \tag{30}$$

$$\bar{V}_{out} = \bar{V}_{Co} = \frac{\bar{V}_p}{(1-\bar{D})} - \bar{I}_O \frac{R_p + R_L + \bar{D}(1-\bar{D})R_{Co}}{(1-\bar{D})^2} \tag{31}$$

$$\bar{V}_{out} = \bar{V}_{Co} = \frac{\bar{V}_p}{(1-\bar{D})} - \bar{I}_O \frac{R_p + R_L + \bar{D}(1-\bar{D})R_{Co}}{(1-\bar{D})^2} \tag{32}$$

$$\eta = \frac{P_{out}}{P_{in}} = \frac{1}{\left(1 + \frac{1}{R_{Load}}\left(\frac{R_p + R_L}{(1-\bar{D})} + \bar{D}R_{Co}\right)\right)} \tag{33}$$

From Equation 32, it is clear that the duty cycle control can force the current to be sinked or sourced from V_{out}.

3.4 Power Flow Control

Figure 5 shows the operation of the switches in the different power flow mode. When power is flowing from the input to the output, switch Q1 conducts. Depending on the type of switches used, either the body diode can be made to conduct or Q2 can be switched ON to enable more efficient power flow, if it can handle current in both direction (like MOSFET). Figure 5 shows the diagram of the inductor current in case of IGBTs and hence Q2 is not switched ON in the case of forward power flow. When the inductor current drops to zero under transition time, the pattern of the pulse width modulation (PWM) is changed and Q1 is no longer switched. For reverse power flow in case of IGBTs, Q2 is switched to control the power flow with a (1-D) duty. Such reconfiguration to the PWM switching pattern is easy to implement using the flexibility of the PWM module on C2000 microcontrollers.

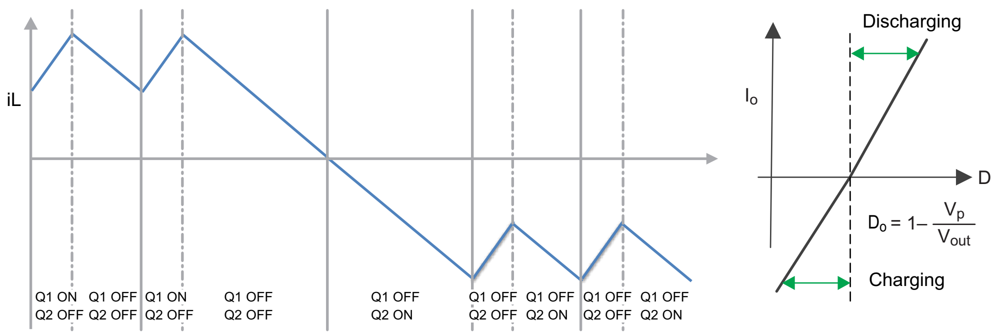

Figure 5. Transition of Power Flow Using Duty Cycle Control

3.5 Small Signal Model

For the small signal model, the delta terms from the large signal model, Equation 16, are aggregated as shown in Equation 34:

$$\dot{\hat{X}} = (A_1\bar{D} + A_2(1-\bar{D}))\hat{X} + (B_1\bar{D} + B_2(1-\bar{D}))\hat{U}_i + ((A_1 - A_2)\bar{X} + (B_1 - B_2)\bar{U}_i)\hat{d} + (A_1 - A_2)\hat{X}\hat{d} + (B1 - B2)\hat{U}_i\hat{d} \tag{34}$$

Ignoring the double multiple of the delta terms and assuming the only applied disturbance is the duty cycle perturbation, the small signal model is written as shown in Equation 35:

$$\dot{\hat{X}} = (A_1\bar{D} + A_2(1-\bar{D}))\hat{X} + ((A_1 - A_2)\bar{X} + (B_1 - B_2)\bar{U}_i)\hat{d} \tag{35}$$

Now assuming $A = A_1\bar{D} + A_2(1-\bar{D})$ and talking Laplace Transform, Equation 36 can be written.

$$\frac{\hat{X}(s)}{\hat{d}(s)} = (sI - A)^{-1}((A_1 - A_2)\bar{X} + (B_1 - B_2)\bar{U}_i) \tag{36}$$

Similarly, the small signal term for the output can be written as shown in Equation 37:

$$\hat{Y} = (C_1D + C_2(1-D))\hat{X} + (C_1 - C_2)\bar{X}\hat{d} + (E_1D + E_2(1-D))\hat{U}_i + (E_1 - E_2)\bar{U}_i\hat{d} \tag{37}$$

Again, assuming the only disturbance in the system is the duty cycle change. The output small signal model can be written as shown in Equation 38:

$$\hat{Y} = C\hat{X} + (C_1 - C_2)\bar{X}\hat{d} + (E_1 - E_2)\bar{U}_i\hat{d} \tag{38}$$

Transfer functions for each output variable control are derived in Section 3.5.1.

3.5.1 Inductor Current to Duty Transfer Function

For inductor current C1=C2=[0 0 1] and E1=E2=[0 0], therefore, the output small signal model is reduced to:

$$\frac{\hat{i}_L(s)}{\hat{d}(s)} = C\frac{\hat{X}(s)}{\hat{d}(s)} = C\left[(sI - A)^{-1}((A_1 - A_2)\bar{X} + (B_1 - B_2)\bar{U}_i)\right] \tag{39}$$

$$\frac{\hat{Y}(s)}{\hat{d}(s)} = \frac{1}{L}\left\{\frac{V_p}{(1-\bar{D})} - \frac{I_o(R_p + R_L)}{(1-\bar{D})^2}\right\}\frac{\left(s + \frac{1}{C_i(R_p + R_{Ci})}\right)\left(s + \frac{I_o}{C_o\left\{\frac{V_p}{(1-\bar{D})} - \frac{I_o(R_p + R_L)}{(1-\bar{D})^2}\right\}}\right)}{\left(s^3 + s^2\left(\frac{R'}{L} + \frac{1}{C_i(R_p + R_{Ci})}\right) + s\left(\frac{R'}{LC_i(R_p + R_{Ci})} + \frac{R_p^2}{LC_i(R_p + R_{Ci})^2} + \frac{(1-\bar{D})^2}{LC_o}\right) + \frac{(1-\bar{D})^2}{LC_oC_i(R_p + R_{Ci})}\right)} \tag{40}$$

$$R' = R_L + (1-\bar{D})R_{Co} + \frac{R_{Ci}R_p}{(R_{Ci} + R_p)} \tag{41}$$

3.5.2 Input Current to Duty Transfer Function

For input current $C1 = C2 = \left[\begin{matrix} 0 & -\frac{1}{(R_{Ci} + R_p)} & \frac{R_{Ci}}{(R_{Ci} + R_p)} \end{matrix}\right]$ and $E1 = E2 = \left[\begin{matrix} 0 & \frac{R_{Ci}}{(R_{Ci} + R_p)} \end{matrix}\right]$ hence, the input current small signal model is reduced to:

$$\frac{\hat{i}_p(s)}{\hat{d}(s)} = C\frac{\hat{X}(s)}{\hat{d}(s)} = C\left[(sI - A)^{-1}((A_1 - A_2)\bar{X} + (B_1 - B_2)\bar{U}_i)\right] \tag{42}$$

$$\frac{\hat{i}_p(s)}{\hat{d}(s)} = \frac{R_{Ci}}{(R_{Ci}+R_p)}\frac{\hat{i}_L(s)}{\hat{d}(s)} + \frac{\frac{R_p}{LC_i(R_{Ci}+R_p)^2}\left\{\frac{V_p}{(1-\bar{D})} - \frac{I_o(R_p+R_L)}{(1-\bar{D})^2}\right\}\left(s + \frac{I_o}{C_o\left\{\frac{V_p}{(1-\bar{D})}-\frac{I_o(R_p+R_L)}{(1-\bar{D})^2}\right\}}\right)}{\left(s^3 + s^2\left(\frac{R'}{L}+\frac{1}{C_i(R_p+R_{Ci})}\right) + s\left(\frac{R'}{LC_i(R_p+R_{Ci})}+\frac{R_p^2}{LC_i(R_p+R_{Ci})^2}+\frac{(1-\bar{D})^2}{LC_o}\right)+\frac{(1-\bar{D})^2}{LC_oC_i(R_p+R_{Ci})}\right)} \tag{43}$$

3.5.3 Output Voltage to Duty Transfer Function

For output voltage $C1$ = [1 0 0], $C2$ = [1 - 0 R_{C0}] and $E1$ = $E2$ = [-R_{C0} 0] and, therefore, the output current small signal model is reduced to:

$$\frac{\hat{V}_O(s)}{\hat{d}(s)} = C\frac{\hat{X}(s)}{\hat{d}(s)} = C\left[(sI-A)^{-1}((A_1-A_2)\bar{X}+(B_1-B_2)\bar{U}_i)\right]+(C_1-C_2)\bar{X} \tag{44}$$

$$\frac{\hat{V}_O(s)}{\hat{d}(s)} = \frac{\left(\left(-\frac{\bar{I}_O}{(1-D)C_O}\right)\left(\left(s+\frac{1}{C_i(R_p+R_{Ci})}\right)\left(s+\frac{R'}{L}\right)+\frac{R_p^2}{LC_i(R_p+R_{Ci})^2}\right)+\frac{(1-D)}{C_O}\left(\frac{V_p}{(1-\bar{D})}-\frac{I_O(R_p+R_L)}{(1-D)^2}\right)\left(s+\frac{1}{C_i(R_p+R_{Ci})}\right)\right)}{\left(s^3+s^2\left(\frac{R'}{L}+\frac{1}{C_i(R_p+R_{Ci})}\right)+s\left(\frac{R'}{LC_i(R_p+R_{Ci})}+\frac{R_p^2}{LC_i(R_p+R_{Ci})^2}+\frac{(1-D)^2}{LC_O}\right)+\frac{(1-D)^2}{LC_OC_i(R_p+R_{Ci})}\right)}$$
$$+(1-D)R_{CO}\frac{\hat{i}_L(s)}{\hat{d}(s)}+V_{CO}-R_{CO}\bar{i}_L \tag{45}$$

4 Simulating the Model

The model can be easily simulated in MATLAB® using the following script.

```
%%%%%%%%%%%%%%%%%%% Boost Modelling for current control %%%%%%%%%%
% Inductor Current,Input Current & Output Voltage Transfer Function
% Texas Instruments
% Digital Control Systems Group, Houston, TX
% Manish Bhardwaj
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%  This software is licensed for use with Texas Instruments C28x
%  family DSCs.  This license was provided to you prior to installing
%  the software.
% -----------------------------------------------------------------
%         Copyright (C) 2010-2014 Texas Instruments, Incorporated.
%                         All Rights Reserved.
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
s = tf('s');

% Input Voltage
Vp=200;
% Input resistance
Rp=2.75*10^-3*200;
% Input capacitance
Cci=1*10^-3;
% Input capacitance ESR
Rci=74*10^-3;
% DCR of the inductor
Rl=9.6*10^-3;
% Inductance
L=130*10^-6;
```

```
% Output Capacitance
Co=15*10^-3;
% Output Capacitance ESR
Rco=5*10^-3;
% Output Load
Io=80;
% Duty cycle
D=0.5;
% Equation 41
Reff=Rl+(1-D)*Rco+(Rci*Rp/(Rci+Rp));

% (sI-A)
SIminusA=[s 0 (-(1-D)/Co);0 (s+1.0/(Cci*(Rp+Rci))) (Rp/(Cci*(Rp+Rci))); ((1-D)/L) (-
(Rp/(L*(Rp+Rci)))) (s+Reff/L)];
% (sI-A)^-1
inv_SIminusA=inv(SIminusA);
% (A1-A2)X+(B1-B2)U
A1minusA2XPlusB1minusB2U=[-Io/((1-D)*Co); 0 ;((Vp/((1-D)*L))-(Io*(Rp+Rl)/(L*(1-D)^2)))];

% Inductor current Trasnfer Function
% Equation 39
% C*((sI-A)^-1*((A1-A2)X+(B1-B2)U))
y_IL=[0 0 1]*(inv_SIminusA*A1minusA2XPlusB1minusB2U);

% Input Current Transfer Function
% Equation 42
% C*((sI-A)^-1*((A1-A2)X+(B1-B2)U))
y_IP=[0 -1/(Rci+Rp) Rci/(Rci+Rp)]*(inv_SIminusA*A1minusA2XPlusB1minusB2U);

% Output Voltage Transfer Function
C1=[1 0 0];
C2=[1 0 Rco];
C=[1 0 (1-D)*Rco];

Il=Io/(1-D);
Vci=Vp-Io*Rp/(1-D);
Vco=(Vp/(1-D))-((Io*(Rp+Rl+D*(1-D)*Rco))/(1-D)^2);
X=[Vco; Vci; Il];
% Euqation 45
% C*((sI-A)^-1*((A1-A2)X+(B1-B2)U))+(C1-C2)*X
y_Vo=C*(inv_SIminusA*A1minusA2XPlusB1minusB2U)+(C1-C2)*X;

y_IL = minreal(y_IL);
zpk(y_IL)

y_IP= minreal(y_IP);
zpk(y_IP)

y_Vo = minreal(y_Vo);
zpk(y_Vo)

figure;
hold on;
bode(y_IL);
bode(y_IP);
bode(y_Vo);
title('Comparison of different transfer functions');
legend('Il/D','Ip/D','Vo/D');


% For digital implementation the plant needs to discretized
%Switching Frequency is 10Khz
Fs=10000;
% Switching time period
Ts= 1/Fs;
y_IL_D=c2d(y_IL,Ts);
```

```
% call sisotool ofr the compensator design
sisotool(y_IL_D);
```

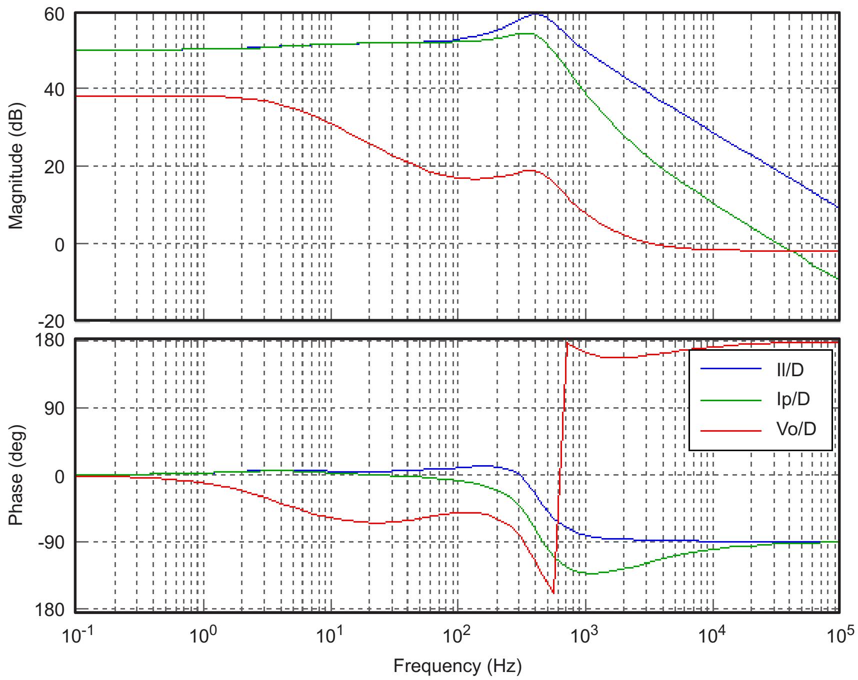

Figure 6. Comparison of Different Control Transfer Functions

Once the design is complete in sisotool, export the compensator into matalab and it will look something like this:

$$\frac{0.003262 - 0.002516z^{-1}}{1 - z^{-1}} \tag{46}$$

You may have to do tf(Comp) to see the transfer function in this form. This type of structure of the compensator can be easily implemented using the C2000 Digital Power Library or the C2000 Solar Library. The structure of the compensator implemented in these libraries is:

$$\frac{U(z)}{E(z)} = \frac{B3z^{-3} + B2z^{-2} + B1z^{-1} + b0}{1 - A3z^{-3} - A2z^{-2} - A1z^{-1}} \tag{47}$$

Observing Equation 46 and Equation 47, the coefficients can be written to be programmed as follows:

B3=0, B2=0, B1= -0.002516, B0=0.003262

A3=0, A2=0,A1= 1

5 Conclusion

This report presents a unified state space model for bi-directional buck/boost converter. All relevant transfer function for inductor current, input current and output voltage control are derived and simulation script provided. Compensation design for digital implementation and how C2000 digital power/C2000 Solar Library Two Pole Two Zero or Three Pole Three Zero blocks can be used to implement the compensation is presented.

6 References

- Bidirectional DC DC Power Converter Design Optimization, Modeling & Control, Junhong Zhang, PHD Dissertation, Virginia Polytechnic Institute, 2008.
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