Application Report
SLOA096 – December 2001

More Filter Design on a Budget

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ABSTRACT

This document describes filter design from the standpoint of cost. Filter design techniques that require the fewest possible op amps and passive components are described. Six types of filters are described—low pass, high pass, narrow bandpass, wide bandpass, notch, and band reject.

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1 Introduction

Why *filter design on a budget*? The answer is self-evident—fewer op amps are less expensive, take up less space on a PC board. Fewer passive components also means less space on a PC board, less parts to stock, less assembly and less test time. It seems that every reference on filter design that the author encountered was written by academics whose love for mathematical derivation was more important than considerations of putting a real design into production. Naturally, all filter topologies are presented in their works without an interpretation of *what is best*. In some applications, it might make sense to use topologies that use three op amps to implement only two poles—perhaps the end equipment is expensive and a few extra components are not a concern. This author suspects that the vast majority of applications are constrained in cost and PC-board space. A telephone handset that uses only one op amp and four passive components to filter speech is less expensive and smaller than one that uses three op amps and 10 passive components for the exact same filter response. These are the new realities of analog design.

The first article in this series – *Filter Design on a Budget* (reference 1) concentrated on passive components—just those values which are really needed to implement a filter with the desired frequency response. This article concentrates on the actual filter topologies. It answers the question, *Without compromising response in any way, how can a filter be implemented using the minimum number of op amps and passive components?* It focuses exclusively on double pole Butterworth response filters, although other filter response characteristics can be accomplished, using the proper design techniques.

2 Low Pass Filter

A low pass filter is used to eliminate high frequency harmonics from an analog waveform. It has a response that extends from dc to a cutoff frequency, which is defined as the point at which the amplitude has declined 70.7% (or 3 dB) from its initial value at low frequencies. The response of a Butterworth double pole low pass filter is shown in Figure 1. After the initial 3 dB attenuation (shown at the red marker), the response at a frequency ten times higher (shown by the blue marker) is down 40 dB (a one hundred times reduction).

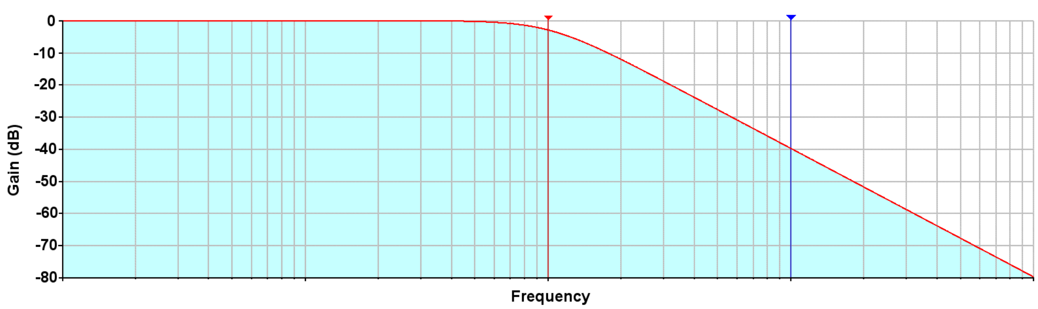

Figure 1. Low Pass Filter Response

In practice, low pass filter response degrades close to dc. Most op amps exhibit a *pink noise* characteristic at low frequencies, which eventually makes the op amp very noisy at very low frequencies (milli- or micro- Hertz). In addition, single-supply op amp circuits employ dc blocking capacitors, which introduce a one-pole high pass characteristic to the response. The designer can place this high pass pole as low in frequency as desired, however.

Truism: There is no such thing as an ac-coupled single-supply active low pass filter. They are bandpass filters with a low frequency cutoff determined by the selection of the coupling capacitor.

There are two very good double pole low pass topologies—Sallen-Key and Multiple Feedback (MFB). Sallen-Key, as shown in Figure 2 is also available in a version with gain, but there is little advantage to it—it adds two additional resistors. The MFB topology can be used for gains of more than one.

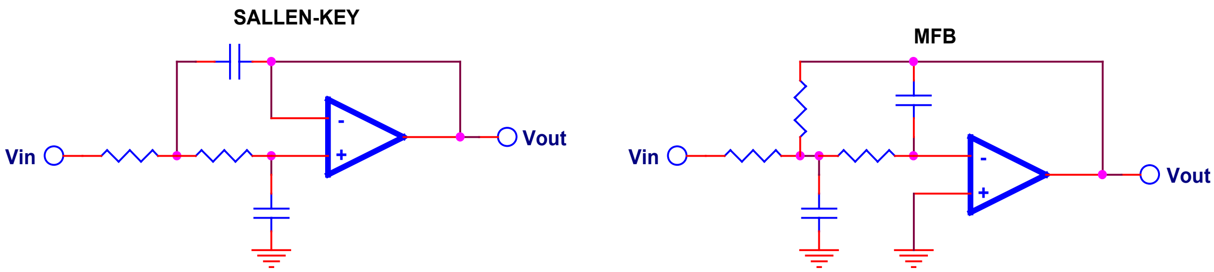

Figure 2. Low Pass Filter

Component count:

Op amp: 1

Capacitor: 2

Resistor: 2 to 3, depending on topology selected

Single Supply Modification:

Single supply modification is easily accomplished for the MFB topology by moving ground connections to half supply, and ac-coupling. It is difficult for the Sallen-Key topology—it uses two additional resistors to create a virtual ground at the input. It is better to use MFB for single supply low pass applications.

Fully-Differential Modification:

Fully-differential modification is easily accomplished by duplicating the feedback path for the MFB topology. It is not possible for Sallen-Key topology.

Design Procedure:

Design procedure is too complex for inclusion here—refer to a textbook on the topic.

Limitations:

The Sallen-Key topology shown above is limited to unity gain. Although it is possible to use two additional resistors with the Sallen-Key topology to provide gain, there is no advantage to doing so. The MFB topology can accomplish the same thing with one less resistor.

3 High Pass Filter

A high pass filter is used to eliminate low frequency harmonics from an analog waveform. It has a response that extends down from infinity to a cutoff frequency, which is defined as the point at which the amplitude has declined 70.7% (or 3 dB) from its initial value. The response of a Butterworth double pole high pass filter is shown in Figure 3. The 3 dB attenuation frequency is shown at the red marker. The response at one-tenth the –3 dB frequency (shown by the blue marker) will be down 40 dB (a one hundred times reduction).

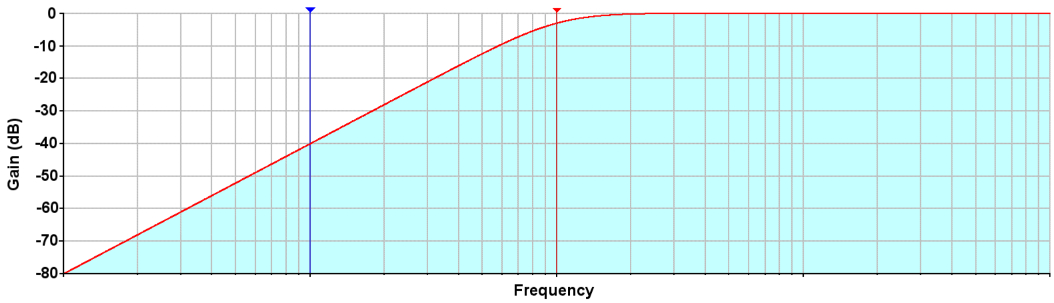

Figure 3. High Pass Filter Response

Figure 3 implies that the filter can pass energy out to infinity. In practice however, high pass filter response does not extend to infinity. Op amps have an ultimate bandwidth limitation, which is the point at which the closed loop response of the op amp intersects the open loop response. This is the gain bandwidth limitation of the op amp, and the response rolls off at –20 dB per decade above this limit, which gives a one-pole low pass response.

Truism: There is no such thing as an active high pass filter. They are bandpass filters with a high frequency cutoff determined by the selection of op amp and gain.

There are two very good double pole high pass topologies—Sallen-Key and Multiple Feedback (MFB). Sallen-Key as shown in Figure 4 is also available in a version with gain, but there is little advantage—it adds two additional resistors. The MFB topology can be used for gains more than one.

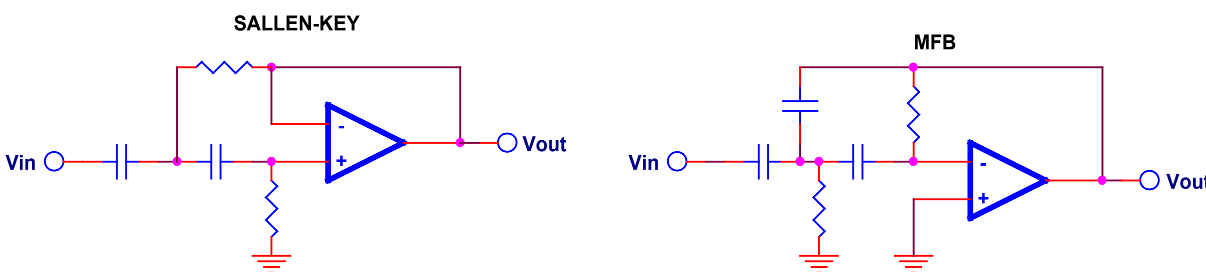

Figure 4. High Pass Filter

Component count:

Op amp: 1

Capacitor: 2

Resistor: 2 to 3, depending on topology selected

Single Supply Modification:

Single supply modification is easily accomplished for either topology. Accomplished by moving ground connections to half supply. AC-coupling is not required at the input; capacitors associated with the high pass topology act to isolate dc potential.

Fully-Differential Modification:

Fully-differential modification is easily accomplished by duplicating the feedback path for the MFB topology. It is not possible for Sallen-Key.

Design Procedure:

Design procedure is too complex for inclusion here – refer to a textbook on the topic.

Limitations:

The Sallen-Key topology shown above is limited to unity gain. Although it is possible to use two additional resistors with the Sallen-Key topology to provide gain, there is no advantage to doing so. The MFB topology can accomplish the same thing with one less resistor.

4 Bandpass Filters

Bandpass filters are used for everything from tone detection to passing a broad range of frequencies. Depending on the bandwidth requirements, these tasks can require completely different design approaches. This application note uses the terms *narrow bandpass* and *wide bandpass*.

The Sallen-Key and MFB topologies have bandpass variations. They place different types of components in impedance locations in a topology. For example, a resistor may be changed to a capacitor in the MFB topology. This serves to take one of the poles of a double pole low pass / high pass variation, and convert it to the other type. A two-pole low pass filter, for example, has one of its poles changed to a high pass pole, leaving one high pass pole and one low pass pole. Similarly, a high pass filter is converted to a bandpass by taking one high pass pole and converting it to low pass, leaving one high pass and one low pass pole.

This *one pole* response characteristic is not the end of the story. From the preceding discussion, the designer would expect only 20 dB per decade rolloff in the stop bands regardless of the Q (sharpness) of the filter. But that is not the case—the transfer function of a bandpass filter forces the response to take whatever slope is necessary to satisfy the gain at the center frequency and the –3 dB points. The slope of the response of high Q bandpass filters can be quite steep near the center frequency. All bandpass filters, however, revert to 20 dB per decade rolloff characteristic away from the center frequency. As the Q becomes lower, the response begins to look more and more like a single pole low pass filter on the low end of the pass band, and a single pole high pass filter on the high end of the pass band.

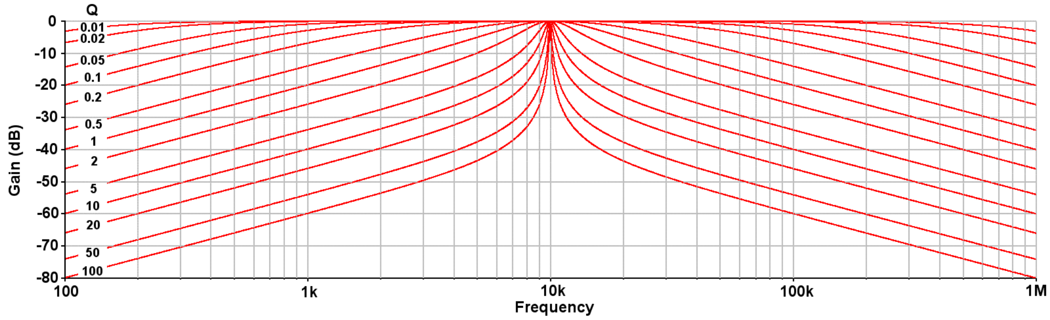

Figure 5. Bandpass Filter Q Comparison

This leads to a question—is it more advantageous to implement a wide band pass by implementing a low pass filter and a high pass filter? If a designer uses cascaded bandpass stages, the best that can be obtained is additional first order rolloff on the low and high end. If the designer concentrates separately on the low and high ends of the band, the result is far superior. Often times, the requirement for rejection on one or the other end of the band is different from the requirement at the other. It may be very stringent for the high frequency end, but the low end of the band may only have the requirement to reject dc (ac-couple). Therefore, it is better to implement low Q bandpass filters as cascaded high pass and low pass filters. The only tricky part for the designer is determining at what point the tradeoff occurs.

The figures to follow show a progression of Q values from 0.1 to 1. The bandpass implementation is shown in red and the cascaded high pass and low pass implementation in blue. Different regions have different shading in the figures, according to the legend in Figure 6:

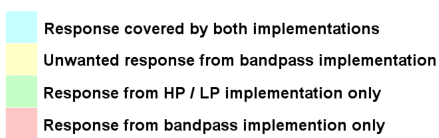

Figure 6. Legend for Bandpass Filter Responses

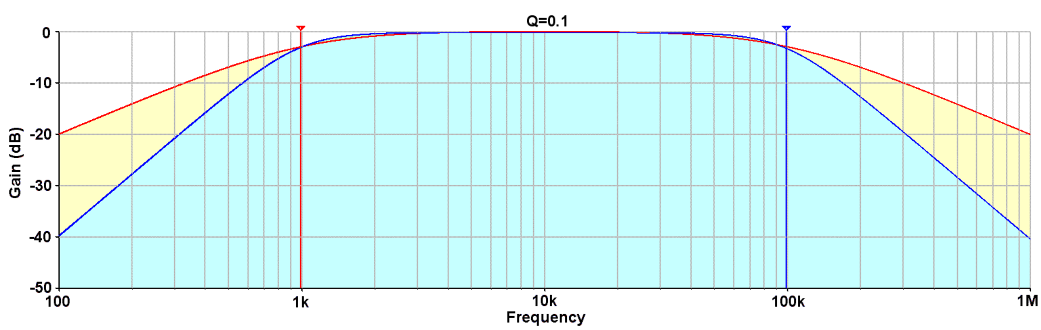

Figure 7. Bandpass vs High Pass / Low Pass, Q = 0.1

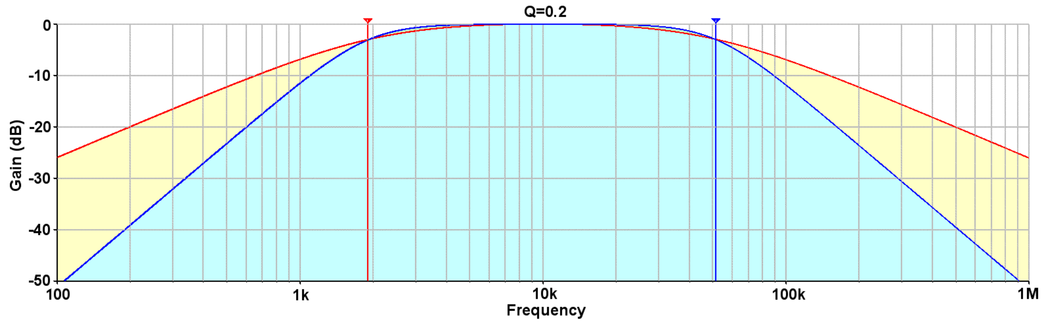

Figure 8. Bandpass vs High Pass / Low Pass, Q = 0.2

Clearly, for Q values of 0.1 (and below), and 0.2, the best implementation is high pass cascaded with low pass. The yellow regions correspond to a large amount of energy in the stop bands that is not rejected with a band pass filter. In the pass band, the cascaded approach is also clearly superior, because there is a wider region in the passband where response is flat.

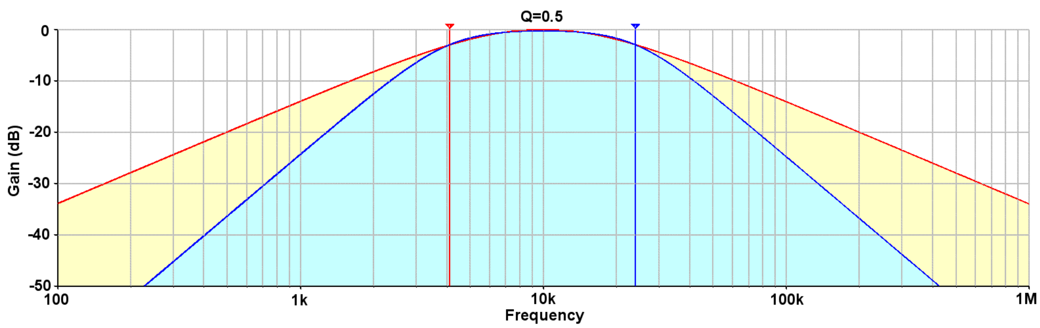

Figure 9. Bandpass vs High Pass / Low Pass, Q = 0.5

The two implementations have almost an identical pass band response for a Q of 0.5. The designer is presented with a choice—use a bandpass filter (which can be implemented with a single op amp) to save money, or use a cascaded approach that has better rejection in the stop bands.

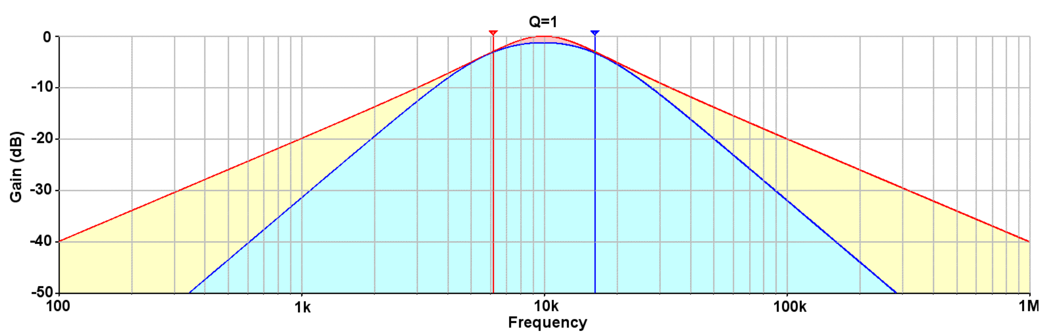

Figure 10. Bandpass vs High Pass / Low Pass, Q = 1

As the Q becomes higher and higher, however, the response of two separate stages begins to interact, destroying the amplitude of the signal. The designer at this point can still opt for the cascaded approach if stop band rejection is primary concern, but amplitude response is secondary. Amplitude response begins to degrade badly for even higher values of Q, however, ending the usefulness of the cascaded approach.

4.1 High Q Bandpass – Modified Deliyannis Topology

A number of filter topologies were considered for this function, such as Twin T, MFB, and Sallen-Key. These were abandoned because of component spread problems, complex algorithms, or other problems. Even the Deliyannis topology is not perfect, but it seems to be the best of the single op-amp bandpass topologies. When modified as shown in Figure 7, it is easy to use.

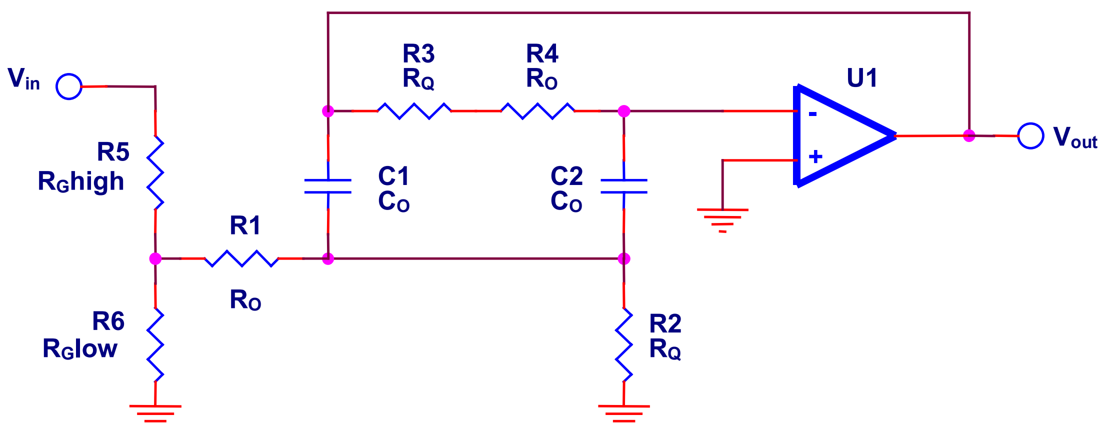

Figure 11. Modified Deliyannis Topology

Component count:

Op amp: 1

Capacitor: 2

Resistor: 3 (if R3 and R4 are combined) to 5, depending on gain and Q needed

Single Supply Modification:

Single supply modification is easily accomplished by moving ground returns to half supply and ac coupling.

Fully-Differential Modification:

Fully-differential modification is easily accomplished by duplicating the feedback path.

Design Procedure:

- The center frequency is determined by the relation:

$$f_O = \frac{1}{2\pi R_O C_O} \quad (1)$$

Where:

$R1 = R4 = R_O$

$C1 = C2 = C_O$.

- Gain and Q are both determined by the expression:

$$\frac{R3 + R4}{2 * R1} = \frac{V_{out}}{V_{in}} = Q \tag{2}$$

Where:

$$n * R3 = R_O = \frac{1}{n} * R2 \tag{3}$$

If R3 is doubled, R2 must be halved and vice versa. If one is tripled, the other must be one third, etc. R2 and R3 must always be related in this way. Otherwise, the center frequency and other circuit characteristics are changed.

- Because Gain and Q are linked together, gain resistors R5 and R6 can be used as a voltage divider to reduce the input level and compensate for this effect. When Gain and Q approach one, short R5 and open R6.

Watch the gain bandwidth product of the op amp carefully for high values of Q. Allow at least 40 dB of safety margin above the peak at the resonant frequency. Also, use an op amp with a high slew rate.

If R1 = R2 = R3 = R4, then Q and Gain are both equal to one.

Limitations:

The circuit cannot be used below a gain and Q of 0.5, because at these values, R3 has to be zero and R2 must be infinite (open). There is no way to boost the gain at Q values less than one, other than to use a separate gain stage. This increases the op amp count by one and the resistor count by two.

4.2 Low Q Bandpass – Cascaded High Pass / Low Pass Topology

The best way of implementing a low Q bandpass filter is to cascade a high pass filter and a low pass filter (in that order). It is preferable to make the high pass stage first, because high frequency noise generated by it can be attenuated in the final low-pass stage.

There are two common single op amp topologies – Sallen-Key and MFB.

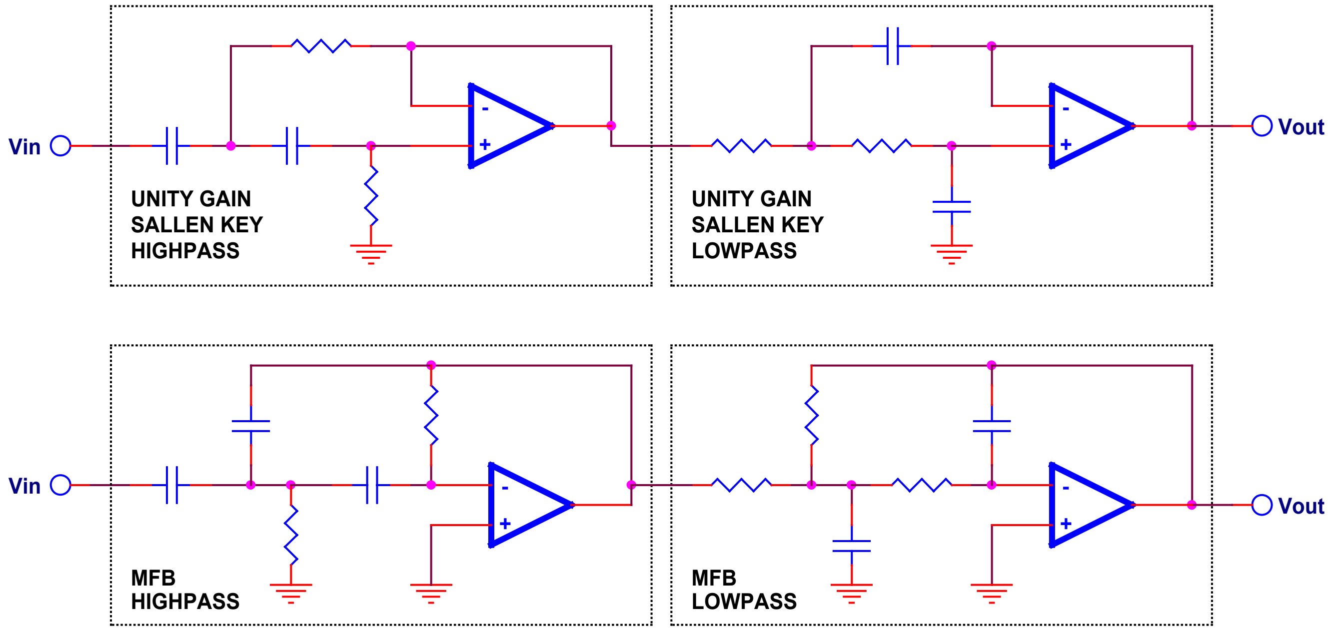

Figure 12. Low Q Band Pass – High Pass Cascaded With Low Pass

Component count:

Op amp:	2
Capacitor:	4 to 5, depending on filter topology selected for low pass and high pass
Resistor:	4 to 5, depending on filter topology selected for low pass and high pass

Single Supply Modification:

Single supply modification is easily accomplished for both implementations by moving ground returns to half supply and ac-coupling.

Fully-Differential Modification:

Fully-Differential modification is possible for the MFB implementation only—by duplicating the feedback path.

Design Procedure:

Design procedure is beyond the scope of this application note, but covered in numerous textbooks and application notes. Design a high pass filter for the lower end of the range, and a low pass filter for the upper end of the range.

Limitations:

- Complex design procedure
- Sallen-Key approach is limited to unity gain with four passive components.

5 Notch and Band Reject Filters

A notch filter is primarily used to reject a single frequency, while a band reject filter is designed to reject a range of frequencies. There are implementations similar to the bandpass case—a true notch corresponding to a narrow bandpass, and a band reject corresponding to a wide band pass.

The depth of the notch for notch filters is largely independent of the Q. Any appearance to the contrary in the figures to follow is an accident of the number of samples used to generate the plot. Q affects the bandwidth of where the –3 dB points lie, which results in a gradually more *washed out* appearance of the notch filter response (blue trace) as shown in the sequence from Figure 14 to Figure 20. Any of the notch filter Q values shown give excellent rejection of the center frequency. If a band of frequencies is to be rejected, however, a notch filter is not the most efficient way to do it. As the Q becomes lower and lower, a lot of excess energy is passed, as shown in the yellow areas of Figure 14 through Figure 20.

The cascaded implementation that worked well for wide bandpass applications cannot be used for wide notch (band reject) filters. This is because the response characteristics have to overlap, or everything becomes attenuated. The only technique that forms a band reject filter is a summed low pass and high pass stage. The response of this configuration is shown in red in Figures 14 to 20. At a Q of 1, it only has a rejection of 7 dB, and is almost useless. It begins to have better rejection as Q values are decreased. At a Q value of 0.05, rejection is over 40 dB—making the summed low pass and high pass implementation clearly superior for band rejection filter. The pink area, however, shows energy near the center frequency that only the notch filter can reject. The designer must decide whether the center frequency rejection is of prime importance, or whether it is better to reject a band of frequencies.

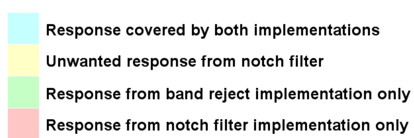

Figure 13. Notch and Band Reject Filter Legend

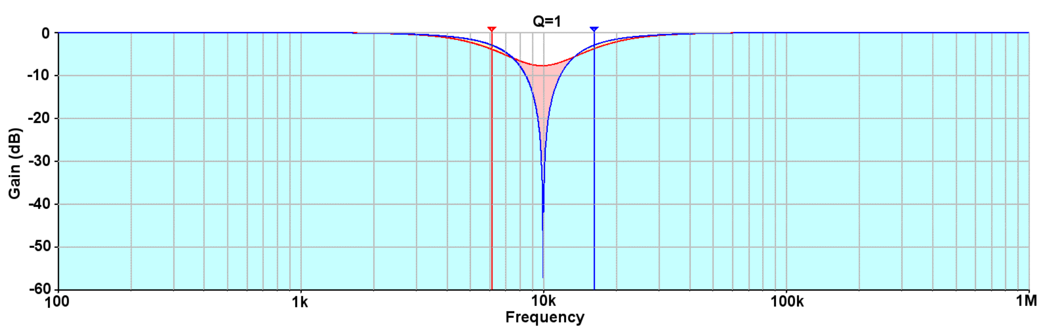

Figure 14. Q = 1, Notch and Band Reject Filter

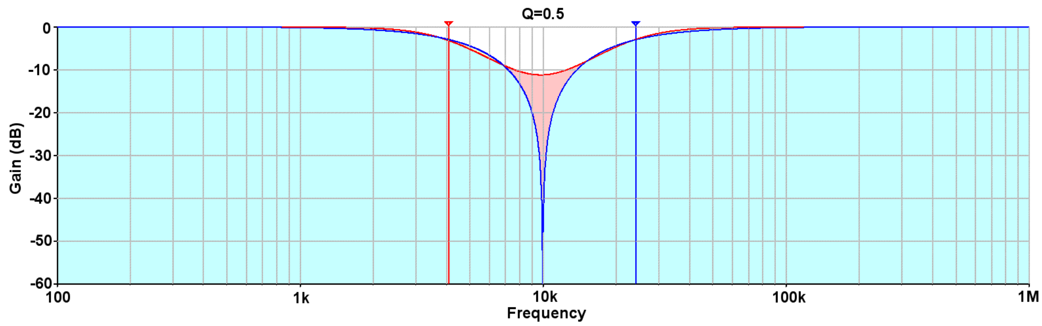

Figure 15. Q = 0.5, Notch and Band Reject Filter

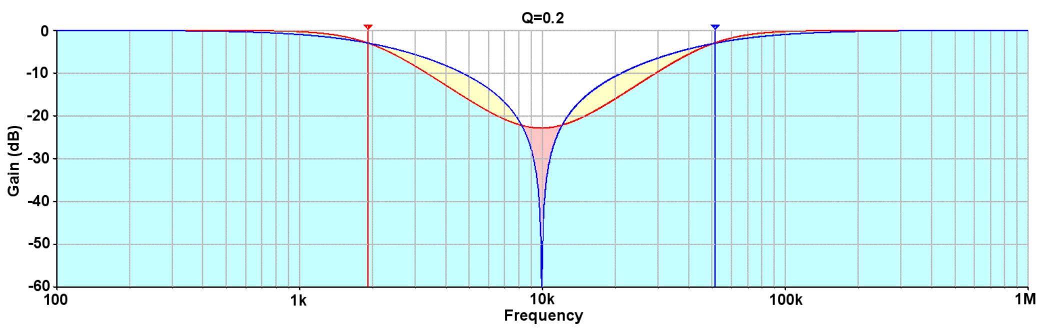

Figure 16. Q = 0.2, Notch and Band Reject Filter

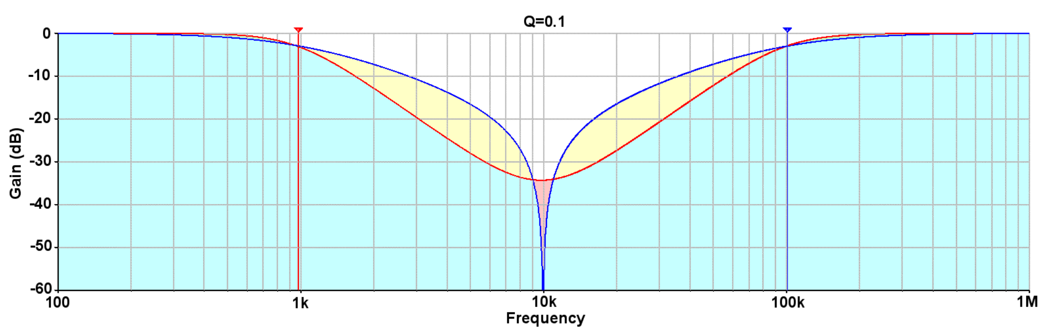

Figure 17. Q = 0.1, Notch and Band Reject Filter

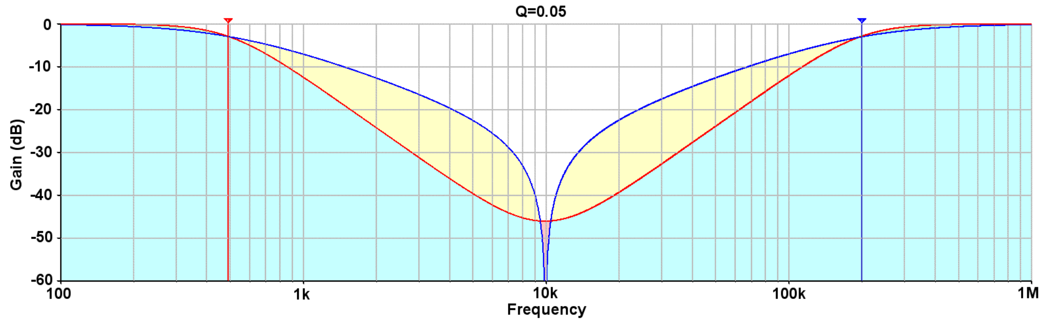

Figure 18. Q = 0.05, Notch and Band Reject Filter

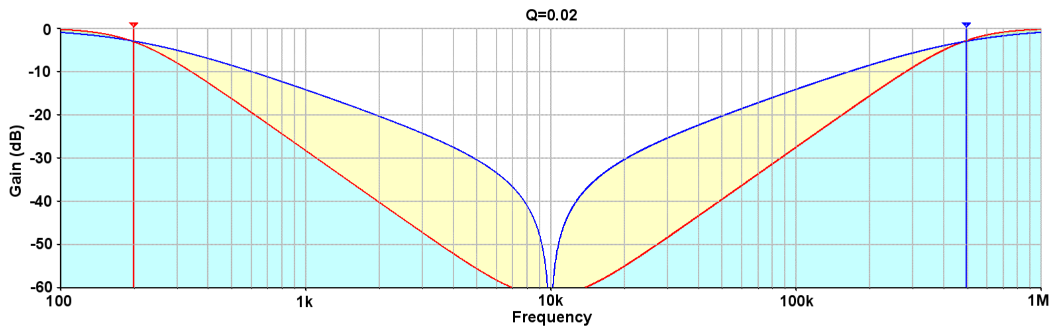

Figure 19. Q = 0.02, Notch and Band Reject Filter

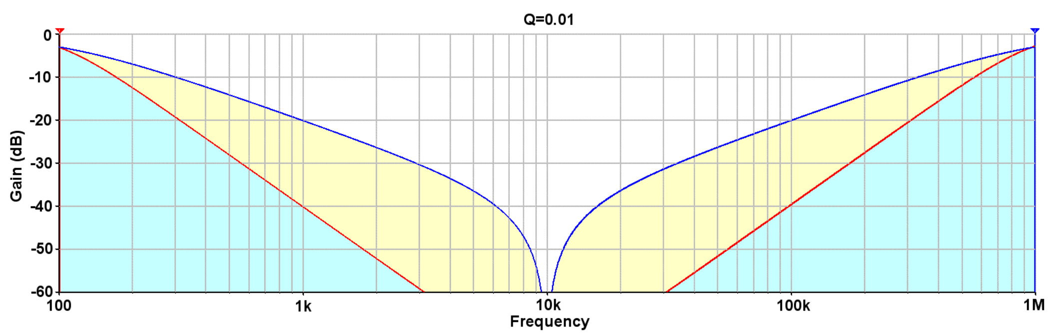

Figure 20. Q = 0.01, Notch and Band Reject Filter

5.1 Notch Filter – Fliege Topology

A number of notch topologies exist. It is very easy to accomplish a good notch filter with three op amps, and not so easy to implement a notch in one or two op amps. This document assumes that notch filters have unity gain, which simplifies things somewhat.

The Sallen-Key and Twin T notch topologies were considered, but reluctantly abandoned as being impractical for one or more of the following reasons:

- There is not a good algorithm that describes the relationship between resistor value and Q.

- The resistors that set the Q become too critical for high values of Q.
- The response is not symmetrical around the center frequency.

Therefore, this document presents the two op amp Fliege topology for notch filter applications.

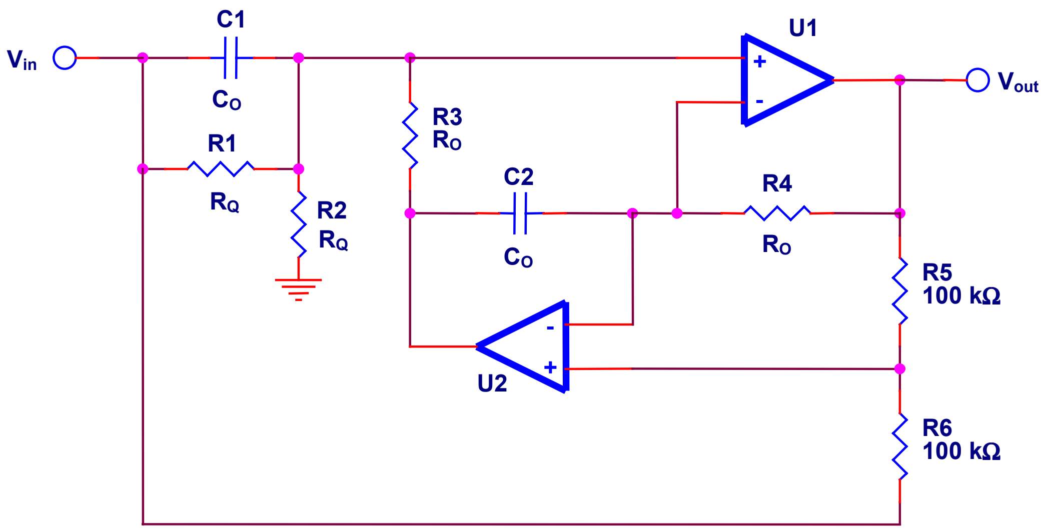

Figure 21. Fliege Notch Filter

Component count:

Op amp: 2

Capacitor: 2 (critical)

Resistor: 4 critical, 2 non-critical

Single Supply Modification:

Connection of R2 to ground is changed to half supply, and capacitively coupled. Watch the common mode range of U2.

Fully-Differential Modification:

Fully-differential modification is not possible.

Design Procedure:

- The center frequency is determined by the relation:

$$f_O = \frac{1}{2\pi R_O C_O} \tag{4}$$

Where:

$R3 = R4 = R_O$

$C1 = C2 = C_O$.

- Q is determined by the expression:

$$R_Q = 2 * Q * R_O \quad (5)$$

R5 and R6 are non-critical, but should be the same value.

Limitations:

- Limited to unity gain

5.2 Band Rejection Filter—Summed High Pass / Low Pass Topology

Band rejection filters are used when a relatively wide band of frequencies need to be rejected. Possible situations would be a switching power supply conversion frequency that changes with load, or harmonics from an unlocked phase locked loop circuit.

Figure 22 shows the implementation of a band rejection filter using summed low pass and high pass stages.

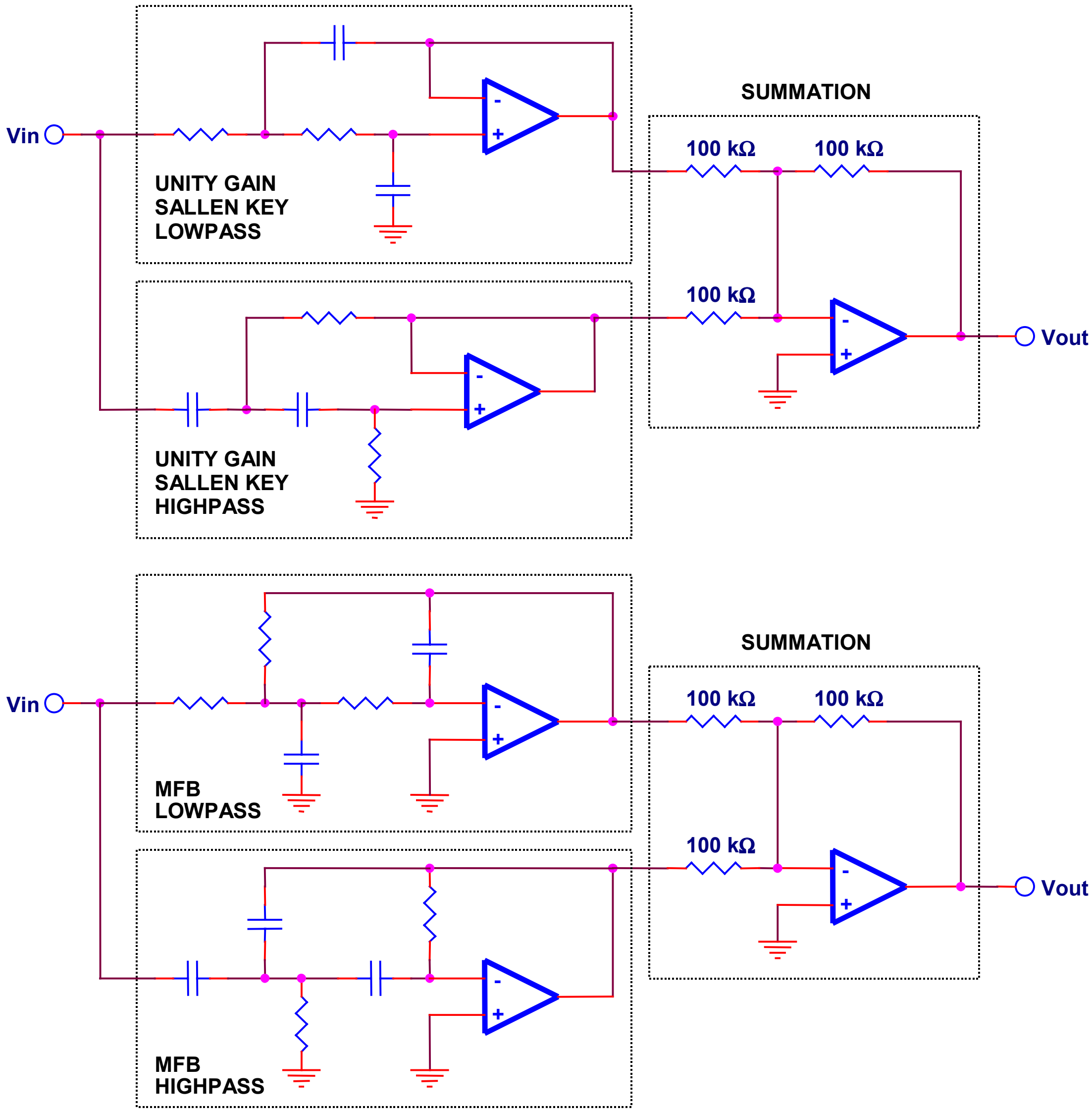

Figure 22. Band Reject Filter Implementations

It should be pointed out that the op amp required by the summation stage may already be present—a buffer op amp for an analog to digital converter, for example. So it may not be a significant increase in cost.

Component count:

Op amp: 3

Capacitor: 4 to 5, depending on filter topology selected for low pass and high pass

Resistor: 6 to 8, depending on filter topology selected for low pass and high pass

Single Supply Modification:

Single supply modification is easily accomplished for both implementations by moving ground returns to half supply and ac-coupling.

Fully-Differential Modification:

Fully-differential modification is possible for the MFB implementation only—by duplicating the feedback path.

Design Procedure:

Design procedure is beyond the scope of this application note, but covered in numerous textbooks and application notes. Design a high pass filter for the lower end of the range, and a low pass filter for the upper end of the range.

Limitations:

- Complex design procedure
- Sallen-Key is limited to unity gain as shown above.

References

1. *Filter Design on a Budget*, Texas Instruments SLOA065

Appendix A—Summary of Filter Characteristics

Table 1. Cost of Implementation

Desired Function	Topology	Op Amp	C	R	Q	Limitations
Low Pass	Sallen-Key	1	2	2		Unity gain
	MFB	1	2	3		
High Pass	Sallen-Key	1	2	2		Unity gain
	MFB	1	3	2		
Narrow Bandpass	Deliyannis	1	2	3 to 6	0.5 to ∞	Gain and Q Interact
Wide Bandpass	Cascaded HP LP SK	2	4	4	< 0.5	Unity gain
	Cascaded HP LP MFB	2	5	5	< 0.5	
Notch	Fliege	2	2	4	0.05 to ∞	Unity gain
Band Reject	Summed HP LP SK	3	4	6	< 0.5	Unity gain
	Summed HP LP MFB	3	5	8	< 0.5	

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