Dealing with nonlinearity in LVDT position sensors

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Introduction

Sense elements convert a physical quantity of interest into electrical signals. One common characteristic of the output of a sense-element is nonlinearity – the output of the a sense element does not vary linearly with the physical quantity of interest. This nonlinearity leads to inaccuracies, or errors, in measurements.

This article describes methods to correct the nonlinearity in the output of linear variable differential transformer (LVDT) position sensors that are used in many applications, including automotive hydraulic-valve position sensing. This discussion also applies to other types of automotive sensor applications such as ultrasonic park-assist.

Sensors with high-frequency outputs

The electrical signal produced by many sense elements is a relatively high-frequency signal. This is either because the stimulus to the sense element is a high-frequency signal, or the physical quantity being measured is high frequency in nature. For example, position measurement of automotive hydraulic valves using LVDT position sensors. This sensor is high frequency because the LVDT primary coil is excited using a high-frequency signal – say, 5 kHz. Similarly, the output of piezoelectric transducers used in ultrasonic park-assist applications is high frequency because the transducers measure the intensity of ultrasonic waves, whose signal frequency is greater than 20 kHz.

In such high-frequency-sensor signal outputs, the signal information is most often embedded in the amplitude of the signal. Figure 1 shows a time-plot of a sensor signal which is high frequency, and the amplitude represents the variation of the physical quantity that the sensor is measuring. The mathematical representation of such a signal is:

$$y(t) = A_C \sin(\omega_C t) + A_S \sin(\omega_S t) A_C \sin(\omega_C t), \qquad (1)$$

where A_C is the amplitude of the sensor excitation, ω_C is the sensor-excitation frequency in rads/s, A_S is the amplitude of the physical quantity of interest, and ω_S is the frequency of the physical quantity of interest in rads/s.

It is assumed that the amplitude of the signal of interest is less than the amplitude of the high-frequency carrier signal so that y(t) is an undistorted amplitude-modulated signal.

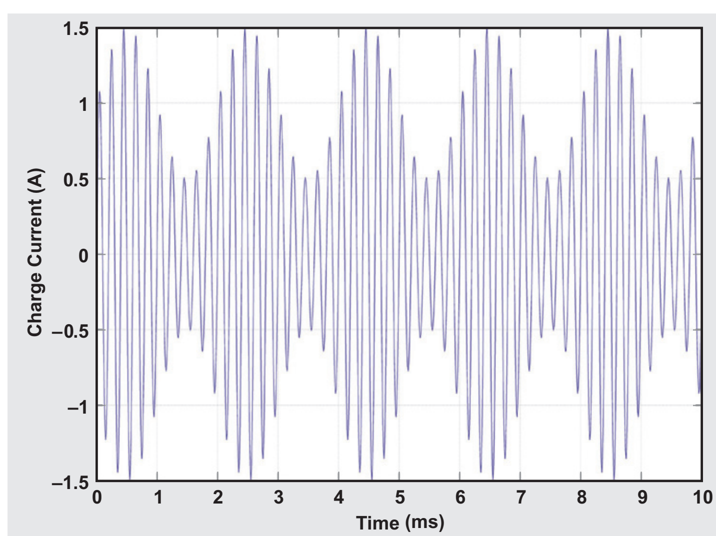
Note that in some cases, the signal information could be embedded in the signal's frequency. For example, if an ultrasonic transducer in the bumper or wing mirror of a moving automobile is used to measure distance to another moving vehicle, then the frequency of the signal will have information about the relative speeds of the vehicle based on the Doppler effect.

In this article, the focus is on extracting the amplitude information from the output of the high-frequency sensor. The technique of extracting amplitude information is called amplitude demodulation.

Amplitude-demodulation techniques have been in existence ever since AM radio transmissions began in the early 1900s. Many solutions have been implemented in both the analog and digital domain to extract the amplitude information from a signal. Further, the demodulation processes could extract just the amplitude of the signal, or could also extract the phase of the sensor output signal relative to the sensor stimulus. The former demodulation technique is called asynchronous demodulation while the latter technique is called synchronous demodulation.

Based on the above definitions, the focus of this article is limited to one specific family of amplitude-extraction techniques – digital asynchronous amplitude demodulation.

Figure 1. High-frequency sensor output with signal information in the amplitude



Why is this narrow family of techniques important? With the advent of advanced manufacturing techniques for mixed-signal integrated circuits (ICs), such as the LBC8LV process at Texas Instruments, mixed-signal signal conditioners are available to support a variety of sensors. With such sensors, signal conditioning occurs partly in the analog domain and partly in the digital domain. Even though signal-conditioning architectures have quantization aspects, such devices offer flexibility and, if designed right, offer sufficient accuracy for most sensor applications. Application engineers can easily customize signal conditioners for their specific application scenario and speed up their product-development cycles.

The PGA450-Q1 from Texas Instruments is an example of a mixed-signal signal conditioner for automotive sensors used in ultrasonic park-assist applications. This signal conditioner specifically amplifies the electrical signal using an amplifier and extracts the amplitude of the signal using the digital asynchronous, amplitude-demodulation technique.

Digital asynchronous, amplitude-demodulation techniques

Two methods of the digital asynchronous, amplitude-demodulation technique are investigated: peak and average.

Peak Method

In this method of asynchronous demodulation, the peak value of the sensor output signal in every frequency cycle is extracted. That is, this process discretizes the signal in time at a frequency of the underlying carrier signal.

If the frequency of the underlying signal is known (which is usually the case), and if the signal is sinusoidal (which is a typical high-frequency excitation signal), then the extracted peak value is the amplitude of the sine wave in each cycle. That is, the peak value in the nth carrier-frequency cycle can be mathematically expressed:

$$y_P\left(t = nT_c\right) = A_C + A_S \sin\left[\omega_S\left(n - \frac{3}{4}\right)T_C\right], \tag{2}$$

where $T_C = 2\pi/\omega_C$.

The PGA450-Q1 implements the peak method of demodulation.

Average Method

In this method of asynchronous demodulation, the sensor signal is first rectified. The rectified output is then filtered using a low-pass filter. Mathematically, the full-wave rectified output of the signal described in Equation 1 is:

$$\begin{aligned} y_R(t) &= \left|y(t)\right| \\ &= \left[A_C \sin(\omega_C t) + A_S \sin(\omega_S t) A_C \sin(\omega_C t)\right] u(t), \end{aligned} \tag{3}$$

where u(t) is a square wave with amplitude equal to 1 and frequency equal to ω_C.

Using Fourier series, the square wave can be written as:

$$u(t) = \frac{4}{\pi}\sum_{n=0}^{\infty}\frac{\sin\left[(2n+1)\omega_C t\right]}{2n+1} \tag{4}$$

Therefore, the rectified output can be written as:

$$y_R(t) = \left[A_C \sin(\omega_C t) + A_S \sin(\omega_S t) A_C \sin(\omega_C t)\right]\left\{\frac{4}{\pi}\sum_{n=0}^{\infty}\frac{\sin\left[(2n+1)\omega_C t\right]}{2n+1}\right\} \tag{5a}$$

$$y_R(t) = \frac{4A_C}{\pi}\sum_{n=0}^{\infty}\frac{\sin(\omega_C t)\sin\left[(2n+1)\omega_C t\right]}{2n+1} + \left\{\frac{4A_S A_C}{\pi}\sum_{n=0}^{\infty}\frac{\sin(\omega_S t)\sin(\omega_C t)\sin\left[(2n+1)\omega_C t\right]}{2n+1}\right\} \tag{5b}$$

$$\begin{aligned} y_R(t) &= \frac{4A_C}{\pi}\sin^2(\omega_C t) + \frac{4A_S A_C}{\pi}\sin(\omega_S t)\sin^2(\omega_C t) + \frac{4A_C}{\pi}\sum_{n=1}^{\infty}\frac{\sin(\omega_C t)\sin\left[(2n+1)\omega_C t\right]}{2n+1} \\ &+ \left\{\frac{4A_S A_C}{\pi}\sum_{n=1}^{\infty}\frac{\sin(\omega_S t)\sin(\omega_C t)\sin\left[(2n+1)\omega_C t\right]}{2n+1}\right\} \end{aligned} \tag{5c}$$

$$\begin{aligned} y_R(t) &= \frac{2A_C}{\pi} - \frac{2A_C}{\pi}\cos(2\omega_C t) + \frac{2A_S A_C}{\pi}\sin(\omega_S t) - \frac{2A_S A_C}{\pi}\sin(\omega_S t)\cos(2\omega_C t) \\ &+ \frac{4A_C}{\pi}\sum_{n=1}^{\infty}\frac{\sin(\omega_C t)\sin\left[(2n+1)\omega_C t\right]}{2n+1} + \left\{\frac{4A_S A_C}{\pi}\sum_{n=1}^{\infty}\frac{\sin(\omega_S t)\sin(\omega_C t)\sin\left[(2n+1)\omega_C t\right]}{2n+1}\right\} \end{aligned} \tag{5d}$$

Filtering the rectified output using a low-pass filter with a gain of 0 dB and the cutoff frequency is ω_C, the filtered output can be expressed:

$$y_{LPF}(t) = LPF[y_R(t)] \cong \frac{2A_C}{\pi} + \frac{2A_S A_C}{\pi}\sin(\omega_S t) \quad (6)$$

Equation 6 shows that the signal can be extracted using this method.

Effect of nonlinearity

In a linear system, the output of the system is proportional to its input; in other words, the output is:

$$y = \sum_{i=1}^{1} a_i x^i, \quad (7)$$

where x is the input to the system, y is the output of the system, and a_i is the coefficient. For simplicity of analysis, the offset term (the term that is independent of the input signal) has been neglected.

In a nonlinear system, the output of the system has higher-order input terms; or the output is:

$$y = \sum_{i=1}^{\infty} a_i x^i. \quad (8)$$

In the context of LVDT signal conditioning, the amplitude modulated signal given in Equation 1 could be affected by nonlinearity. The possible causes of nonlinearity are:

1. Distorted drive signal, or the carrier has higher-order harmonics. The carrier could have high-order harmonics because the carrier signal itself is the output of a nonlinear system:

$$y_C = \sum_{i=1}^{\infty} a_i [A_C \sin(\omega_C t)]^i$$

2. The nonlinear signal chain, or the signal output, is nonlinear with regards to its input:

$$y_S = \sum_{i=1}^{\infty} a_i [A_C \sin(\omega_C t) + A_S \sin(\omega_S t) A_C \sin(\omega_C t)]^i$$

3. Nonlinear transducer, or the transducer output, is nonlinear with regards to the physical quantity it is measuring:

$$y_S = \sum_{i=1}^{\infty} a_i [A_S \sin(\omega_S t)]^i$$

Note that the first two sources of the nonlinearities are a result of a non-ideal signal generator/conditioner, while the third source of nonlinearity is from the transducer.

Also, all nonlinearities can be present in the system at the same time, making the output of the signal chain a complex mathematical expression.

Dealing with nonlinearity

The two sources of nonlinearity that are a result of non-ideal signal generator and conditioner will now be addressed. Also, a second-order nonlinear system, which is one of the common forms of nonlinear systems, will be analyzed. This analysis can be extended to higher-order nonlinearities as well as a nonlinear transducer output.

Distorted drive signal

In the presence of a distorted drive signal (or carrier signal), the amplitude-modulated carrier signal given in Equation 1 can be written as Equation 9 by setting $a_1 = 1$ and $a_2 = b$ after trigonometric manipulations.

$$y(t) = A_C \sin(\omega_C t) + A_S \sin(\omega_S t) A_C \sin(\omega_C t) + b[A_C \sin(\omega_C t)]^2 + bA_S \sin(\omega_S t)[A_C \sin(\omega_C t)]^2 \quad (9a)$$

$$y(t) = A_C \sin(\omega_C t) + A_S \sin(\omega_S t) A_C \sin(\omega_C t) + bA_C^2[1 - \sin(2\omega_C t)] + bA_S A_C^2 \sin(\omega_S t)[1 - \sin(2\omega_C t)] \quad (9b)$$

Equation 9 shows that the output of the transducer has frequency components at 0 rad/s and at around $2\omega_C$, in addition to a signal around ω_C

One clear way to minimize the frequency components at 0 rad/s and at around $2\omega_C$ is to use a bandpass filter with the center frequency set at ω_C and with sufficient bandwidth. The bandwidth specifically should be such that there is no significant attenuation at $\omega_C \pm \omega_S$. With such a bandpass filter, the output of the bandpass filter is:

$$y_{BPF}(t) = BPF[y(t)] \cong A_C \sin(\omega_C t) + A_S \sin(\omega_S t) A_C \sin(\omega_C t) \quad (10)$$

This bandpass filter output can then be demodulated either by using the peak or average method to extract the transducer signal.

Nonlinear signal chain

The presence of second-order signal-chain nonlinearity causes the amplitude modulated signal to be modified to be:

$$y(t) = \sum_{i=1}^{2} a_i [A_C \sin(\omega_C t) + A_S \sin(\omega_S t) A_C \sin(\omega_C t)]^i \quad (11)$$

Setting a1 = 1 and a2 = b, and following trigonometric manipulation, Equation 11 can be written:

$$
\begin{aligned}
y(t) = & \left\{\left[A_C \sin(\omega_C t) + A_S \sin(\omega_S t) A_C \sin(\omega_C t)\right]\right\} + b\left\{\left[A_C \sin(\omega_C t)\right]^2\right\} \\
& + b\left\{\left[A_S \sin(\omega_S t) A_C \sin(\omega_C t)\right]^2\right\} + b\left[2A_C \sin(\omega_C t) A_S \sin(\omega_S t) A_C \sin(\omega_C t)\right]
\end{aligned} \quad (12a)
$$

$$
\begin{aligned}
y(t) = & \left[A_C \sin(\omega_C t) + A_S \sin(\omega_S t) A_C \sin(\omega_C t)\right] + bA_C^2\left[1 - \sin(2\omega_C t)\right] \\
& + bA_C^2 A_S^2\left[1 - \sin(2\omega_C t)\right]\left[1 - \sin(2\omega_S t)\right] + 2bA_C^2 A_S\left[1 - \sin(2\omega_C t)\right]\sin(\omega_S t)
\end{aligned} \quad (12b)
$$

$$
\begin{aligned}
y(t) = & \left[A_C \sin(\omega_C t) + A_S \sin(\omega_S t) A_C \sin(\omega_C t)\right] + bA_C^2\left[1 - \sin(2\omega_C t)\right] \\
& + bA_C^2 A_S^2\left[1 - \sin(2\omega_C t) - \sin(2\omega_S t) + \sin(2\omega_C t)\sin(2\omega_S t)\right] + 2bA_C^2 A_S\left[\sin(\omega_S t) - \sin(2\omega_C t)\sin(\omega_S t)\right]
\end{aligned} \quad (12c)
$$

Equation 12, once again, shows that the output of the transducer has frequency components at 0 rad/s and at around $2\omega_C$ and higher frequencies, in addition to the signal around ω_C

Again, using a bandpass filter with the center frequency set at ω_C, and with sufficient bandwidth, the nonlinearity-induced frequency components can be reduced. With such a bandpass filter, the output of the bandpass filter is:

$$
\begin{aligned}
y_{BPF}(t) &= BPF\left[y(t)\right] \\
&\cong A_C \sin(\omega_C t) + A_S \sin(\omega_S t) A_C \sin(\omega_C t)
\end{aligned} \quad (13)
$$

This bandpass filter output then can be demodulated either using the peak or average method to extract the transducer signal.

Conclusion

The output of a LVDT position sensor could be nonlinear. This article described how the use of a bandpass filter in the signal chain can be an effective way to deal with signal nonlinearities.

Figure 2 shows a simplified block diagram of the LVDT signal conditioner described in this article. Specifically, the block diagram shows the use of the bandpass filter in the signal chain.

Note that the PGA450-Q1 signal conditioner from Texas Instruments is designed for automotive ultrasonic park-assist sensors and already implements a bandpass filter.

References

1. Alan Oppenheim and Ronald Shafer, “Digital Signal Processing,” Prentice Hall, 1975

Related Web sites

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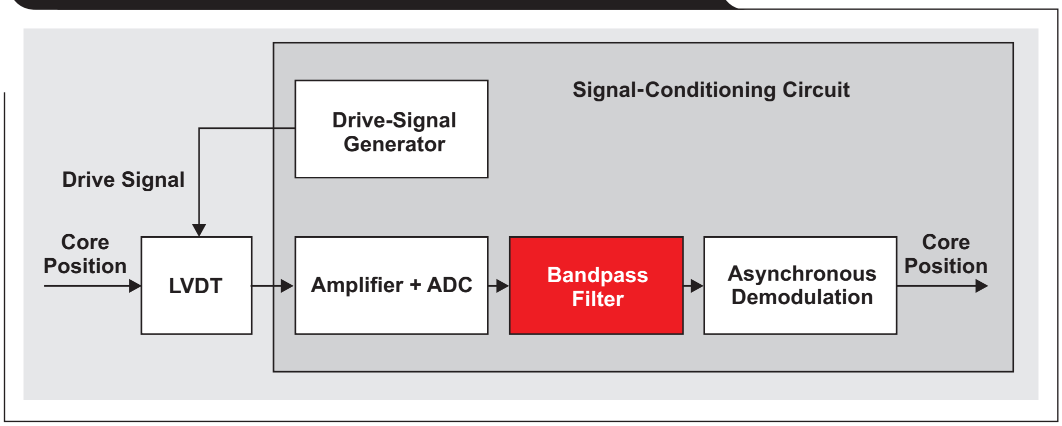
Figure 2: LVDT signal conditioner with bandpass filter



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